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Analytic Aposteriority and its Relevance to Twentieth-Century Philosophy

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Abstract:

This article begins with an overview of the fourfold epistemological framework that arises out of Kant's distinctions between analyticity and syntheticity and between apriority and aposteriority. I challenge Kant's claim that the fourth classification, analytic aposteriority, is empty. In reviewing three articles written during the third quarter of the twentieth century that also defend analytic aposteriority, I identify promising insights suggested by Benardete (1958). I then present overviews of two 1987 articles wherein I defend analytic aposteriority, first as a classification highlighting the epistemological status of several crucial (and easily misunderstood) features of Kant's own philosophy, and second as a way of expressing some of Kripke's claims about naming in more authentically Kantian terminology. The paper concludes with suggestions of several other important philosophical developments that also make advances precisely insofar as they expound the nature and implications of the epistemological classification that Kant assumed to be empty.

1. The Boundary of Knowledge: Kant's Framework of Epistemological Classifications

One of the central features of Kant's ground-breaking *Critique of Pure Reason* is its introduction of a new framework for classifying propositions according to their epistemological status, based on two dyadic distinctions: first, between propositions that evince an "analytic" structure and those with a "synthetic" structure; and second, between "a priori" modes of justifying such propositions and "a posteriori" modes.¹ This gives rise to four possible kinds of propositional knowledge-claim, two of which are relatively non-controversial: analytic a priori propositions establish logical knowledge, whereas synthetic a posteriori propositions establish *empirical* knowledge. As is well known, Kant used one of the two controversial types to demonstrate why Hume's distinction between "matters of fact" (cf. synthetic a posteriori propositions) and "matters of reason" (cf. analytic a priori propositions) does not encompass all possible options. In rejecting the legitimacy of the law of causality, Hume had failed to notice that some (albeit, rare) propositions exhibit a synthetic (factual) structure, yet can be justified through an entirely a priori mode of argument.

Kant's own way of defining this fourfold distinction has been examined by so many past commentators that a thoroughgoing overview of its various nuances would require a book length work. Instead of scrutinizing the interpretive history of this distinction, I shall offer an outline of what are widely accepted to be Kant's basic parameters for understanding each relevant term. Throughout this discussion we must keep in mind that, once again, the first pair of terms refers to the structure of propositions, whereas the second pair refers to their *justification*. A neglect of this difference has given rise to misleading portrayals of synthetic apriority in particular.² However, we need not examine those departures from Kant's approach here, as our interest lies elsewhere.

Kant argued that the structure of a proposition must be such that *either* its predicate is contained within the subject and is therefore self-evident or its predicate lies outside the subject, so that we must appeal to something else in order to ascertain its truth. Propositions of the former type

(e.g., “White is a color”) are *analytic* because we can derive the predicate merely through a logical analysis of the subject; they are not informative, for they tell us only what we already know, assuming we understand the meaning of the words being used. (If I merely say “white”, anyone who understands the word would be likely to assume I am referring to a color, especially if we are in the presence of something white and the listener knows I am referring to that thing). Propositions of the latter type (e.g., “This paper is white”) are *synthetic* because we must appeal beyond the concepts themselves, to what Kant called “intuitions” (or sensible input), in order to ascertain their truth; such propositions are informative, inasmuch as they tell us about some factual state of affairs. Whereas typical examples of analytic propositions carry with them (as in a deduction) a form of conceptual truth that is *necessary*, typical examples of synthetic propositions advance claims (as in an induction) that are *contingent* and therefore might cease to be true, if the facts happen to change. (The paper these words are printed on might have faded into a pale yellow—or for that matter, they might now be appearing on a computer monitor, perhaps with a light blue background).

The second distinction, between a priori and a posteriori modes of justification, seems at first to be coextensive with the first pair, but Kant insists their ranges of application are distinct. A proposition is a priori if we do not need to *appeal* to any particular experience to justify its truth, whereas establishing the truth of an a posteriori proposition requires such an appeal. Obviously, the above examples of analytic and synthetic propositions would also be a priori and a posteriori, respectively, since those examples illustrate the two uncontroversial members of the fourfold distinction. Yet Kant argues that a previously-neglected alternative, the synthetic a priori, is not only possible but constitutes the epistemological status of the most significant truth-claims in all of philosophy. His most famous example, the proposition “Everything that happens has its cause” (CPR B13), is (not coincidentally) the very principle that Hume had downplayed as a groundless “habit” of thought. In Kant’s hands, it becomes a necessary condition (hence, a priori) for all experience of objects (hence, synthetic). Because the appeal here is to experience *in general*, not to any *particular* experience, Kant famously argued that such affirmations of “transcendental” knowledge have the status of absolute (apodictic) certainty: they define the very boundary-conditions that make empirical knowledge possible.

In discussing the key features of this epistemological framework, Kant notes in passing that one of the four logically possible classifications that arises out of this fourfold distinction is simply empty (CPR B11): “*Experiential judgments, as such, are one and all synthetic*. For to base an analytic judgment on experience would be absurd, because in its case I can formulate my judgment without going outside my concept, and hence do not need for it any testimony of experience.” Aside from this lone, off-hand comment, Kant never considers the possibility that this fourth type of proposition might describe a legitimate area of philosophical inquiry. At first sight, he appears to have been correct to rule out the possibility of analytic aposteriority, for if we *must appeal to experience* in order to justify the truth of a given proposition, how can its truth be *grounded entirely in the concepts*? Indeed, if we judge from the extensive secondary literature on this question, then Kant was right. For out of the thousands of scholars who have commented on aspects of this distinction over the past two centuries, only a handful have explicitly questioned Kant’s rejection of this elusive fourth classification.

Despite the almost deafening lack of attention that has been given to the possibility of locating meaningful analytic a posteriori truth-claims, I have previously argued that some of the most interesting features of Kant’s own philosophical system, *as well as* some of the most important advances made by twentieth-century philosophy, can be regarded as a direct outworking of precisely this (admittedly paradoxical) classification. Having offered in this opening section a brief introduction to Kant’s definitions of the key terms, I shall proceed in §2 to discuss three initial attempts that were made during the third quarter of the twentieth century to rescue analytic aposteriority from Kant’s charge of emptiness. While two of those early attempts are of little help, because they were based on misunderstandings of Kant’s original distinction, the other one presents arguments that foreshadow the position I shall defend here, though in a somewhat different way. In §3 I turn to a summary and elaboration of the claim I have advanced in various previous

publications,³ that analytic aposteriority is actually a *crucial* epistemological classification for philosophers to consider, both for a complete understanding of Kant's own philosophical system and for an accurate assessment (within a Kantian framework) of why various contemporary developments in philosophy are so significant. Finally, in §4 I will survey several twentieth-century applications of this elusive classification, arguing that some of the greatest philosophical achievements in recent decades can actually be understood more deeply if they are interpreted as examples of analytic aposteriority.

2. Some Early Attempts to Restore Kant's Suppressed Fourth Classification

Throughout the second half of the twentieth century, especially in the wake of Quine's influential critique [7], the analytic-synthetic distinction fell into considerable disrepute, especially among philosophers in the (somewhat ironically named) "analytic" tradition.⁴ Doubts about the notion of a priori knowledge were quick to follow.⁵ As a result, many philosophers to this day consider the status of so many propositions to be difficult if not impossible to pin down as either analytic or synthetic, and the whole notion of apriority to be so counter-intuitive, that the Kantian framework tends to be discarded as altogether worthless. To counter such objections, I have argued (in "APK") that the "gray areas" that apply to most (if not all) examples of real-life propositions cause problems for the Kantian framework *only* when the four key terms are used in ways that diverge from Kant's guidelines, or when we fail to consider a proposition's *context*. What I call a "perspectival" interpretation of Kant's epistemological framework maintains that any given proposition might, in principle, take on any of the four possible classifications, depending on its use. But before explaining (in §4) how this works as a defense against Quine's critique and how it highlights what is arguably the central error of Kant's philosophical system, let us examine other, earlier attempts to justify the claim that analytic aposteriority may be a meaningful epistemological classification for some types of proposition.

In this section I shall examine three independent attempts, during the third quarter of the twentieth century, to resurrect the notion of analytic aposteriority from the graveyard of implausibility to which Kant consigned it. I have been unable to locate *any* response or even citation to any of the three articles to be discussed here, so there is no need to present them in chronological order. Instead, I shall begin with the least substantive and most problematic article and progress to the one that most successfully explains how and why the analytic a posteriori has a proper place not only in epistemology, but in metaphysics as well.

Virgil Aldrich's attempt to awaken Kantians from the "dogmatic slumber" of merely assuming that analytic aposteriority is self-contradictory [1] consists of a short and comparatively simple argument. What "has kept viable the dogma of there being no analytic a *posteriori* propositions", he claims, is the assumption that analytic *containment* must be *conceptual* (200). He correctly observes that many ordinary propositions are uttered while the speaker is in direct perceptual contact with the subject of the proposition (e.g., "This paper is white"—if you are now actually reading this article printed on white paper). If such perceptual containment counts as analytic just as much as conceptual containment does, then it is a short step to the conclusion that analytic a posteriori concepts are commonplace. After all, one must obviously *experience* the paper (look at it) in order to recognize that the white percept is contained in the paper (201). Unfortunately, this argument draws its entire force from a complete neglect of the crucial fact that for Kant any such requirement that we appeal to a percept (i.e., to what Kant calls an "intuition") *makes the proposition synthetic*. All of the propositions Aldrich thinks he has demonstrated to be analytic a posteriori would therefore, given Kant's definitions, merely be examples of the least controversial of all the classifications: the synthetic aposteriority of ordinary empirical knowledge. In order to be analytic a posteriori on *Kant's* definitions of the terms (see §1), the knowledge that the paper is white *could not come from the percept*, but would have to be attached necessarily to the subject of the proposition *in a completely conceptual manner*. This is not just a dogmatic assumption; it is a crucial defining feature of the fourfold distinction that cannot be amended without radically changing the nature of what is being claimed.

D. Goldstick's attempt to make sense out of Kant's rejected epistemological classification [4] is only slightly longer and slightly more informative than Aldrich's. Fortunately, the misunderstanding that plagues its argument is not so serious. Correctly insisting that "the term 'analytic' ought not be understood as equivalent to 'provable by unaided logical deduction'" (531), Goldstick's aim is "to justify the reasonableness of asserting the *probable* existence of some analytic a posteriori truths" (534, my emphasis). His ensuing argument rests on a rather curious strategy. First, he lists four propositions that each express *potentially* analytic a posteriori truth, because each starts with: "It is logically possible that [or for]..." (532). He then defends the key claim, that "all truths which assert the existence of logical possibilities" are analytic (533), with two reasons: (A) whatever determines logical necessity must also determine what *lacks* logical necessity, and negating the latter "determines also which [propositions] are logically possible" (533); and (B) "logically necessary" is equivalent to "true in all logically possible worlds", so anything that is *actually* (i.e., in our world) logically possible will be logically possible in all logically possible worlds (533). The only issue that remains, then, is whether "belief in the existence of a logical possibility may sometimes be rationally founded" (534). Goldstick defends the *probability* that some analytic a posteriori truths exist by citing three factors that would ground the rationality of such a belief (534): (i) "the existence of a logically valid deductive proof" that has as its conclusion either that some proposition is logically possible or that its negation cannot be logically deduced; (ii) "the existence of empirical evidence" that "no logically valid deductive proof has to date been found for the [latter] negation"; and (iii) "the existence...of [real] empirical evidence for the actual truth of the [former] proposition"—all quite plausible claims. While Goldstick presents an ingenious argument, his conclusion's appeal to probability renders it too weak to be applicable to a Kantian framework, where *apodictic certainty* is the ultimate goal. Moreover, he never clearly explains the crucial distinction between analytic aposteriority and the far less controversial type of logical truth, analytic apriority, thus giving rise to the suspicion that he has actually been dealing with probabilities of the *latter* type all along.

By far the most substantive of the three early attempts to restore credibility to the analytic a posteriori was the first, an article by José Benardete [2], who writes (503):

It is our present object to show that there are in fact analytic truths which are derived from a precise examination of experience, that these truths must be understood as a *posteriori* rather than as a *priori*, and that they are material, rather than merely formal, in their content. In establishing the analytic a *posteriori*, we seek to provide a kind of organon propaedeutic to metaphysics itself.

Benardete goes so far as to claim that, if he is correct, then "[t]he logical question preliminary to metaphysics" must be expressed in precisely the "obverse fashion" (504) of the way Kant expressed it—namely, as "how is the analytic a posteriori possible?" In support of this claim, Benardete appeals to two crucial aspects of our empirical knowledge of the external world, sight and sound, focusing most of his attention on the latter.⁶

Concerning sound, Benardete asks us to consider three basic components of any sound (especially noticeable in an analysis of music): pitch, timbre, and loudness (504-505). These basic components seem to be essentially different from other, more specific characteristics of a sound, such as its "middle-C-ness". With Kant's definition of analyticity as *containment* in mind, Benardete then argues (505-506):

In general, whenever the predicate of a non-identical analytic proposition cannot be subtracted from the subject so as to leave a residue, or if, in some sense, it can be subtracted (as loudness from middle-C-ness) but the residue itself entails that predicate, then we are confronted with a real, and not a nominal, analytic proposition. By means of this method of subtraction, we are equipped with a touchstone or canon by which to certify the analytic a *posteriori*.

A nominal analytic proposition would be a priori: justifiable with reference to nothing more than the meanings of the words. But in the type of proposition Benardete has in mind, where the subject

is “middle-C-ness”, we would have no idea that this subject requires or *contains* some degree of loudness, except by examining the way the empirical world works.

As another example Benardete mentions Hume’s discussion of the hypothetical discovery of a new shade of blue, then asks: “How do we know of this missing shade? By experience? Certainly.”⁷ The proposition that this new shade of blue is a color is just as analytic as the proposition “White is a color”, yet the only way to *discover* (and thus, to justify) the truth of the proposition that *this new shade* is blue is through *experience*. We shall return to this point in §4, when we see how similar arguments were advanced by subsequent twentieth-century philosophers. For now it will suffice to note that Benardete’s argument rests on the claim that *definition* can operate in two distinct ways: as either synthetic or analytic. Thus, comparing sound and color, he distinguishes between these two senses of defining a term or determining a percept (511-512):

The simple sound itself must be described or defined in terms of a definite loudness and timbre as well as a definite pitch; just as the simple color “emerald” must be defined...as yellow-green in hue, of medium saturation, and medium brightness. It has long been fashionable to assume that simple sensations are indefinable. This is quite false.... It is a grave error to look for the indefinables in the names of simple sensations.... Just as no such thing as a bare animal can exist, unspecified as to its being canine, feline, or some other; so, too, a bare hue, which is not some definite hue, cannot exist.... The attributes of a simple sound cannot be dismissed as merely nothing at all. They exist in their own derivative way, as modes. At bottom, there is a certain indeterminacy (or synthetic quality) in simple impressions, as well as a definite determinacy (or analytic quality.)

The status of propositions that are a posteriori, because their justification requires an appeal to something perceived through our five senses, reflects this two-sided situation: if the a posteriori aspect cannot be determined or defined without going beyond the subject-concept to the direct experience of what the predicate describes, then it is the ordinary, synthetic variety; if we can determine or define the applicability of the predicate as being already contained in the subject-concept, due to the *type of experience* under consideration (e.g., due to the way sounds or colors operate), then those claims (e.g., “Every sound has a pitch” or “Every color has a hue”) are analytic even though they, too, are knowable only a posteriori.⁸

Perhaps one of the main reasons Benardete’s insightful arguments fell on rocky soil, at this point in the development of twentieth-century Anglo-American philosophy, is that, as he openly admits, this way of understanding the nature of color and sound would “oblige us to return to a doctrine of real essences” (512)—a metaphysical assumption that was widely discredited at the time.⁹ Nearly all of the *facts* we learn, when we experience percepts such as colors or sounds, will still count as empirical (i.e., synthetic a posteriori), if we accept Benardete’s argument; but the discovery that every experience of color necessarily “possesses the properties of hue, brightness, saturation, figure, and magnitude” (512) must be recognized as analytic a posteriori; for “these latter properties, unlike the former, do exist in a substantial unity, an intelligible necessary connection binding them all together. It is that necessary connection which provides the metaphysical ground for the logical concept of the analytic a posteriori.”

The general rule for determining the epistemological status of a proposition, on Benardete’s view, is to ask, for any proposition that is not a tautology (513),

can the predicate be subtracted from the subject so as to yield a residue, as the predicate “wicked” of the analytic proposition “all ogres are wicked” can be subtracted from the subject “ogre” to yield the residue “giant.” If that subtraction cannot be performed, then the proposition is analytic a posteriori.

In order to appreciate the full force of Benardete’s conclusion, we must consider the other three options that he here leaves unspecified, perhaps because he thought they were too obvious. First, he must be assuming (unproblematically) that tautologies, or nominally analytic propositions (such as “White is a color”), are analytic a priori. What is rather curious is that he says nothing about the status of a proposition for which such a subtraction *can* be performed, but with a residue, as in the very example he provides. His choice of example suggests, though, that he is assuming that such

propositions are also analytic a priori, for his example follows the same structure as the standard example of an analytic proposition, “All bachelors are unmarried”, where subtracting “unmarried” yields the residue “man”. If, instead of canceling out the subject altogether or changing it significantly, the subtraction of the predicate leaves the subject (as such) essentially unchanged, then the proposition is (by definition) *synthetic*: subtract “white” from “paper” in “This paper is white”, and nothing *essential* to the subject actually changes. Presumably, Kant would say the same about subtracting “cause and effect” from “changes” in the archetypal synthetic a priori proposition, “*All changes occur according to the law of the connection of cause and effect*” (CPR A/B232).

Once we see how the three main options in Kant’s epistemological framework fit in with Benardete’s “subtraction” procedure, we can appreciate the force of his claim that, if such a subtraction cannot be performed in a given proposition, then that proposition is analytic a posteriori. The three other classifications are all expressed in terms of predicates that can be subtracted from their subjects, either not changing the subject in any essential way (and thus, synthetic) or else changing it essentially (and thus, analytic). Benardete’s claim is that some propositions *cannot be subjected to this procedure*, and these are the ones Kant left unaccounted for by treating the fourth classification as empty. Surprisingly, Benardete never provides an actual example of a specific analytic a posteriori *proposition*; but from his argument, we may assume that he had in mind the two proposed above: “Every sound has a pitch” and “Every color has a hue.” In each case, the predicate designates a feature of the subject that, *through experience alone*, we recognize as *contained within the concept* of the subject. As we shall see in §4, this focus on what Kripke later called rigid designation, came to be the basis for what was arguably the most important application of analytic aposteriority in the twentieth century. But before we assess such recent applications, let us take a step back and examine what happens to Kant’s own philosophy, if we allow analytic aposteriority to have its proper place.

3. The Need for Analytic Aposteriority in Kant’s Philosophy

Once the analytic a posteriori is recognized as a non-empty member of Kant’s epistemological framework, the question arises as to whether or not Kant’s own philosophy contains propositions with such a status, propositions that would in that case tend to seem out of place or ill-defined as Kant presents them. My earliest work on this topic defends just such a claim, that the architectonic unity of Kant’s own system cannot be fully appreciated apart from an awareness of the role played by the analytic a posteriori.¹⁰ In this section I shall therefore present a summary and further elaboration of that initial application. While I first located analytic aposteriority only in the Dialectic of the first *Critique* and in the Analytic of second, I shall here suggest that it also plays a crucial role in the theory of symbolism defended in the third *Critique* and applied in Kant’s *Religion within the Bounds of Bare Reason*.

In the Dialectic of *CPR*, Kant develops some of the most elaborate and influential arguments in his entire *corpus*, demonstrating that propositions formerly believed to convey genuine metaphysical truth (and therefore serving as prime candidates for the classification he had earlier introduced as being synthetic a priori) are at best inconclusive, and at worst, vacuous. Having completed his demonstration that traditional (“speculative”) metaphysics provides no valid synthetic a priori propositions,¹¹ he concludes the Dialectic with a lengthy Appendix, arguing that the same ideas of reason that *fail* to attain a synthetic a priori status (i.e., God, freedom, and immortality) nevertheless have a *legitimate* function in metaphysics, as *regulative* (rather than constitutive) principles, guiding our search for unity in the systematic ordering of human knowledge. He is careful to caution that, when we view an idea of reason in this way, we are acting *as if* it is true, rather than justifying its truth as a confirmed item of knowledge as such. Later, in Chapter I of *CPR*’s Doctrine of Method, Kant also discusses the role of *hypotheses* in reason’s proper metaphysical employment: philosophers may rightly use hypotheses “as weapons of war” (CPR A777/B805), even though we must treat the concepts they affirm as *beliefs* rather than as objectively confirmed knowledge.

The single most problematic feature of these affirmations of a more promising approach to metaphysics, both in the Dialectic and in the Doctrine of Method, is that, having demonstrated that metaphysics contains no synthetic a priori knowledge, Kant never assigns *any* epistemological status to the crucial counterweight to his rejection of traditional metaphysics. The claim in my early work on this aspect of Kant philosophy (see note 10) was that Kant's whole discussion of the regulative ideas of reason would have been far more convincing, its overall role in the Transcendental Doctrine of Elements would have been more clear, and its intimate connection with the use of hypothetical (belief-centered) reasoning more evident, had he presented his new *path* to metaphysics as one that treats the ideas of reason as analytic a posteriori truth-claims. Assigning a distinct epistemological classification to his new approach would have clearly set Kant's response to Hume (i.e., his defense of the principle of causality) apart from his main (and quite different) project of elaborating a *moral* approach to metaphysics.

What does it mean to assign an analytic a posteriori status to Kant's attempts to rescue the ideas of reason from ultimate meaninglessness? Claiming that a metaphysical proposition that uses a concept of reason (i.e., an idea), such as "God", is analytic means precisely what Kant argues throughout the Dialectic, regarding each of the three ideas: first, *we cannot have any intuition of the object* that such a concept (presumably) refers to; rather, "the hypothetical use of reason aims at the systematic unity of the cognitions of understanding" through the pure concept of *totality*, thus producing a conceptual "unity" that "is *the touchstone of the truth* of the understanding's rules" (CPR A647/B675). Assigning an a posteriori status to this use of reason means that we can know nothing about how to justify propositions about God, freedom (or the universe as a whole), and immortality (or the soul), apart from treating them *as if* they shed light on some specific *experience(s)* that the propositions refer to or imply. (This, for example, is why Kant regards the physico-theological—i.e., the teleological—argument for God's existence as being far more effective than the ontological or cosmological arguments.) Kant sometimes comes so close to stating precisely these features of reason's use of ideas that it is quite remarkable that he failed to recognize their analytic a posteriori status. He says at one point (A311/B367), for instance, that "although no actual experience ever fully attains to that cognition [conveyed by a "concept of reason"], yet any actual experience always belongs to such a cognition." That experience "(perhaps even the whole of possible experience or of its empirical synthesis)" is part and parcel of every idea (i.e., every concept of reason) makes it *a posteriori*. Yet we cannot actually experience the object referred to by that concept, as such; the most we can do is to become aware that all our experiences "belong to" or are *contained within* that idea, thus making it analytic.

Given Kant's own explicit appeal in the Doctrine of Method to the crucial role played by hypotheses in this way of thinking, I have proposed the convention of referring to reflection that aims at establishing analytic a posteriori truth as adopting the *hypothetical* perspective. Just as synthetic apriority defines the *transcendental* perspective that establishes the fundamental boundary-conditions for all the synthetic a posteriori knowledge that we generate from the empirical perspective, so also the analytic aposteriority of the hypothetical perspective establishes the fundamental parameters for all the analytic a priori truth that we verify from the logical perspective. The difference between the hypothetical and logical perspectives is that the latter presents us with a completed whole that can be grasped *without experiencing it*, whereas the former, with its appeal to the *containment of experience* within a pure concept (i.e., an idea), always presents us with a *task* to be completed—a theme that pervades both the Dialectic and the Doctrine of Method and explains why the hypothetical perspective leads naturally to the practical standpoint.

Within CPR's Doctrine of Method, Kant explains that, although this hypothetical perspective (i.e., the mode of reflection that he should have identified as having an analytic a posteriori status) is problematic when assessed from the theoretical standpoint, it nevertheless gives rise to an entirely appropriate application, from the practical (or moral) standpoint. The crucial connection between the hypothetical perspective of theoretical reason and reason's practical standpoint is obscured, as I have argued in KSP 132-137, by the fact that Kant portrays moral metaphysics as somehow providing reason with synthetic a priori principles, even though the

theoretical *Critique* had proved this to be impossible. My perspectival interpretation of Kant's Critical System shows that each *Critique* is based on a distinct standpoint and that each of these standpoints is formed by focusing on one of the four perspectives that guide the development of the argument within each *Critique* (i.e., the transcendental, logical, empirical, and hypothetical perspectives, respectively). What is being critiqued in *CPR* is the attempt to use the understanding alone (i.e., the analytic apriority of the logical perspective) to solve problems that *go beyond* the realm that is the understanding's true home (i.e., the synthetic aposteriority of the empirical perspective) and in so doing to draw (alleged) inferences about the nature of reality as such (i.e., the analytic aposteriority of the hypothetical perspective).

In the second *Critique* the focus changes: whereas the theoretical standpoint takes the understanding (especially in its logical, analytic a priori employment) as its defining perspective, the practical standpoint takes reason (in its hypothetical employment) as its defining perspective. Only within the context of this radical change of standpoint (such that the analytic a posteriori—in this case, the idea of *freedom*—is no longer the conclusion, but the starting-point of the inquiry) can practical reason's search for (practically!) synthetic a priori principles be understood. Interestingly, when Kant distinguishes action in *nature* (i.e., as viewed from the empirical perspective) from *moral* action, he says "this ought expresses a possible action whose basis is nothing but a mere concept" (*CPR* A547/B575); that is, morality occurs when we hypothetically view our experience (a posteriori) as *contained* in a concept such as "good". Taken in this way, we can regard Kant's overall moral philosophy as a defense of the analytic aposteriority of *freedom*.¹² Morality for Kant just is the adoption of a concept as a hypothesis that a person then imposes onto his or her experience in such a way that *experience conforms itself to the concept, rather than vice versa*. This is the essence of the Kantian analytic a posteriori as it appears in his *Critiques*, though without being named.

Recognizing the crucial role of the analytic a posteriori in Kantian (moral) metaphysics brings added focus and clarity not only to the intricacies of his moral philosophy as such, but also to his use of the moral themes in various other applications, such as to the areas of aesthetics and religion. Without going into detail here, I shall mention just one example that relates to both of these areas. Kant's portrayal of beauty as a *symbol* of morality¹³ makes little (or no) sense if we regard it as an expression of any of the three epistemological classifications that Kant explicitly recognizes (i.e., analytic apriority, synthetic apriority, and synthetic aposteriority). Kant himself clearly distinguishes the use of symbols to elucidate *ideas* that have no intuitive instantiation from the use of schemata to elucidate concepts that can be directly manifested in intuition. Only the latter would count as synthetic a posteriori judgments that are grounded in synthetic a priori principles, thus justifying a person in claiming to express objective knowledge through propositions that relate given intuitions to their conceptual features. The function of symbolism, according to Kant, is precisely to fill the gap left by the inadequacy of the three well-established epistemological classifications. When employing a symbol, we employ the faculty of imagination to interpret a set of intuitions stemming from our (a posteriori) experience of some empirical object(s) *as if* they were "contained in" the concept of an idea whose object lies beyond all possible experience. Had Kant recognized that this analogical containment makes the products of the imagination's hypothetical employment *analytic a posteriori*, his appeal to beauty as a bridge between freedom and nature in the third *Critique*, as well as his use of very similar logic in his discussion of religious symbolism (see note 13), would have been much easier to grasp.

4. Naming, Imagining, and The Power of Belief in Twentieth-Century Philosophy

Rather than tracing other aspects of Kant's system that can be interpreted as defending truth-claims of an analytic a posteriori type, I shall turn my attention now to the various ways this epistemological classification can be found operating in twentieth-century philosophy. We have already seen (in §2) that several abortive attempts were made, during the third quarter of the twentieth century, to rescue analytic aposteriority from oblivion. While none of those succeeded in sparking renewed interest, and even my own effort to show that the classification has a place (both

in Kant's system and in philosophy as a whole) has not prompted a flood of responses,¹⁴ some influential advances that have been made in twentieth-century philosophy can be regarded as relating to just this type of truth-claim. In this concluding section I shall therefore examine several examples of the latter, before reflecting briefly on two more recent attempts and on the potential for future development.

Foremost among these new developments are the revolutionary insights about the nature of "designation" (i.e., *fixing a reference*) in general and of naming in particular, elaborated in Saul Kripke's influential book, *Naming and Necessity* [5]. He convincingly defends the existence of two previously neglected classifications of truth, the necessary a posteriori and the contingent a priori (NN 38), based on considerations revolving around the process of naming and the discovery of new facts about the objects so named. Two of his most widely discussed examples, "Hesperus is Phosphorus" (140) and the designation of a meter as being rigidly fixed by reference to the length of a specific stick in Paris (54), illustrate these two unusual epistemological and (as Kripke claims [e.g., 35]) *metaphysical* classifications. As I argued in detail 25 years ago (in "APK"), the conclusions Kripke reaches are largely correct, but his assumption about how they require a revision of Kant's epistemological framework is seriously flawed. His error on the latter is rooted in the fact that Kripke adopts definitions of his key terms (especially "necessary" vs. "contingent", but also "a priori" vs. "a posteriori" and "analytic" vs. "synthetic") that were commonplace among analytic philosophers of his day, but differ in significant ways from Kant's own definitions of the same terms. Once the differences in definitions are accounted for and appropriate translations are made, Kripke's insights turn out to be entirely consistent with Kant's epistemological framework – *provided* we extend Kant's framework to include the analytic a posteriori, as proposed in §3.

In a nutshell, Kant's framework (as summarized in §1) takes analyticity-syntheticity and apriority-aposteriority to be the fundamental distinctions, and interprets necessity-contingency as a subordinate classification that has applications of different types for different classifications of truth. By contrast, Kripke takes necessity-contingency and apriority-aposteriority as basic and interprets analyticity-syntheticity in terms of these classes. In order to translate Kripke's conclusions into Kant's framework, we must in most cases read Kripke's "necessity"/"contingency" as referring to one aspect of what Kant would call "apriority"/"aposteriority", respectively, and his "apriority"/"aposteriority" as equivalent to Kant's "analyticity"/"syntheticity". Even though Kripke's official definition of "analytic" makes analytic aposteriority *impossible*, he comes very close at one point to acknowledging a role for what would be equivalent to this classification, given his own definitions. As noted in "APK" 270n.

Kripke's framework disallows the analytic a posteriori by definition, since "analytic" is stipulated to mean that which is "both necessary and a *priori*" (NN, 39). He admits at one point, however, that his definition of analyticity may be too strict, in which case something very much like the analytic a posteriori is suggested: "If statements whose a priori truth is known via the fixing of a reference are counted as analytic, then some analytic truths are contingent" (NN, 122n, emphasis added).

Applying the proposed mapping of Kripke's terminology onto Kant's, I argued (in "APK" 264, 268-269) that propositions such as "Hesperus is Phosphorus" are not necessary a posteriori (on Kant's terms) but are *either* synthetic a posteriori (if the context concerns the empirical assertion that two apparently very different observed objects are, in fact, the same object) or analytic a priori (if one is attending to the logical meaning of the two names, understood as *both referring to the planet Venus*). Similarly, I argued (in "APK" 265, 269-270) that Kripke's demonstration that propositions rigidly designating a referent are contingent a priori would, according to Kant's framework, amount to a proof that we are using an analytic a posteriori proposition every time we designate in this manner—a feature that is most obvious when we *name* someone (or something) for the first time.

Without going into further detail on Kripke's revolution—the reader interested in its relevance to my defense of analytic aposteriority should consult "APK"—let us note that his

genuine advance on Kant was to demonstrate the crucial difference between *naming* an object and *defining* a term (“APK” 171):

To name requires that we adopt a *practical* perspective, according to which we act “as if” (or stipulate that) a certain object is to be rigidly designated by a certain word. That is, we subsume an object as experienced (a posteriori) under a given concept (analytically). To define, by contrast, requires that we adopt a *logical* perspective, according to which we devote all our attention to accumulating a set of properties which describe a concept uniquely. That is, we subsume a set of general characteristics (a priori) under a given concept (analytically).

What I added to that revolution, in “APK”, was the proposal that Kant’s (or for that matter, Kripke’s) framework for classifying types of truth should be regarded not as establishing fixed categories, but as delineating different *contexts of understanding* a given proposition. In other words, one and the same proposition (such as “fire is hot”) might function in an analytic a posteriori way in one context (e.g., when expressing one’s first discovery that fire is hot), in a synthetic a posteriori way in another context (e.g., when describing one’s experience of a hot fire to others who know already that fire is hot), and in an analytic a priori way in yet another context (e.g., when talking about the meaning of the relevant words). That theory (or something like it) provides the most effective response both to Quine’s criticism of the analytic-synthetic distinction, as well as to the various doubts that have been expressed regarding the usefulness of the distinction between a priori and a posteriori (see reference 7 and note 5).

A decade after “APK” appeared, Andrew Cutrofello published an entire book, *Imagining Otherwise*, explicitly basing his main argument on the proposal that analytic aposteriority constitutes a legitimate epistemological classification.¹⁵ Using Kant’s transcendental philosophy as a sounding board, he presents a “metapsychology” focused on Freudian psychoanalysis in the wake of Lacan. Rather than affirming a perspectival interpretation of Kant’s framework that has room for all four classifications, however, Cutrofello treats the analytic a posteriori and the synthetic a priori as *mutually exclusive*, so that one must *choose* either Kant and the synthetic apriority of transcendental philosophy *or* Freud and the analytic aposteriority of “metapsychological epoché” (IO 3). When the latter option is carried to its completion in the spirit of Hume¹⁶ and various postmodern theorists, what emerges closely resembles the *structure* of Kant’s first *Critique*—*IO* has sections detailing the Aesthetics, Logics, Principles, Paralogisms, Antinomies, Ideals, and Ethics of the unconscious—yet its content consists of an innovative phenomenology of neurosis, perversion, and psychosis. Just as Kant’s emphasis on the importance of synthetic apriority leads him (almost neurotically¹⁷) to impose what amounts to a “taboo against thinking the analytic a posteriori” (141), Cutrofello reads Freud’s “Kantian inheritance” as “a systematic challenge to Kant’s thesis concerning the synthetic a priori” (8-9).

While the details of Cutrofello’s application of analytic aposteriority are intriguing, to say the least, what matters most for our purposes is that Cutrofello has explicitly taken on the challenge of treating this almost forgotten classification with the seriousness it deserves. Indeed, the importance of his study is not so much the specific details of his postmodern critique (or [psycho]analysis) of Kant, as the general fact that he takes as his philosophical backdrop the Continental tradition, encompassing the trajectory from phenomenology and existentialism to deconstructionism and critical theory. As such, his book makes an ideal contrast to Kripke’s, aptly illustrating the relevance of analytic aposteriority to *both* major twentieth-century philosophical traditions. What is ironic about Cutrofello’s approach is that its either-or strategy lends itself to a psychoanalytic diagnosis of irrational exclusivism not unlike the one he levels against Kant. As a result, he ends up offering little more insight than Kant does into the deep epistemological distinction between the synthetic a priori and the analytic a posteriori. By contrast, adopting a perspectival strategy, whereby all four classifications are allowed their proper domains of application, has the potential to lead us beyond the kind of us-and-them labeling that tends to plague any form of exclusivism.

Although I have given but two examples of twentieth-century philosophers who have affirmed the legitimacy of the analytic a posteriori (one implicitly, the other explicitly), many other *potential* applications could be cited. Using the foregoing examples as inspiration, let me therefore conclude by sketching a few of the other areas where twentieth-century philosophy can be fruitfully interpreted as following this line of development. I shall follow the suggestion offered in an essay on the lasting influence of Kant and Kierkegaard (see note 3), where I argued that the impact of these two philosophers on the twentieth century can best be understood in terms of the *interplay* between synthetic apriority and analytic aposteriority. That is, I firmly reject Cutrofello's assumption that these two classifications are somehow locked in competition, forcing us to choose one or the other.¹⁸ For Kant's explicit aim, as he tells us in *CPR* Bxxx, was not only "to annul *knowledge*" of metaphysical ideas (via the synthetic a priori), but also "to make room for *faith*" in those same ideas (via the analytic a posteriori).¹⁹ Along these lines, my suggestion in "PRKK" 256-258 was that the analytic a posteriori shows itself most notably in areas of human experience characterized by the *power of belief*.

The nature and function of belief, as opposed to knowledge, was the focus of vast amounts of attention by analytic philosophers in the twentieth century. Perhaps this emphasis was nowhere manifested more powerfully than in Wittgenstein's *Philosophical Investigations*, where the whole strategy of analyzing ordinary language usage could be fruitfully interpreted in terms of attending to the way our *experiences* of the world (a posteriori) are already contained (analytically) in the meanings of key philosophical words. What the later Wittgenstein accomplished, as a much more significant advance on Kant than that of the early Wittgenstein (whose *Tractatus* focuses more on the interplay between analytic apriority and synthetic aposteriority), was to recognize the open-endedness of human language as the locus of metaphysical power—even if that power is all-too-often misused by philosophers to create problems that are not genuine. Wittgenstein's decision to ground his arguments in the metaphor of a "language *game*" might seem unfortunate; yet this very metaphor can be interpreted, from what I have called Kant's hypothetical perspective (with its inevitable outworking in the practical standpoint), as the foundation for the analytic aposteriority of his whole approach: our linguistic structures are to be viewed analytically, with attention focused on the meanings *contained within* them, yet that very containment is to be explained by examining the experienced reality of the "game" that forms the context of their usage.

Tracing the Continental tradition back to its earliest roots in Husserl's *Logical Investigations* and Heidegger's *Being and Time*, we can detect similar potential applications for a perspectival understanding of analytic aposteriority. Cutrofello's appeal (though only in passing) to the concept of epoché (see *IO* 3)—what Husserl also called "bracketing"—suggests the significance of this notion to phenomenology; as a further development of (and *complement* to) Kantian transcendental reflection, it can be interpreted as a call to view the nature of reality in the open-ended terms of analytic aposteriority. Similarly, the role of intentionality and imagination in demarcating a "horizon" of understanding, from which context each person sets out to interpret the world, can best be understood as a conceptually-grounded power (hence, analytic) that encompasses within it each experience a person calls his or her own (hence, a posteriori).²⁰ Admittedly, these are only hints as to how one might proceed in relating this much-maligned epistemological classification to the developments of twentieth-century philosophy; but in a nutshell, much of the emphasis of both analytic and Continental philosophers during the past century has been on unveiling the depths of experienced reality in ways that go *beyond* what Kant called "empirical knowledge", so my point here is merely that a clearer awareness of the *status* of such post-empirical knowing (as analytic a posteriori) would provide a fruitful way of understanding how these developments relate to other, more time-honored aspects of human knowing.

Finally, although my main focus in this paper has been on the twentieth century, it is worth mentioning that two (admittedly, rather meager) attempts have been made, during the first decade of this century (both in 2003), to restore respectability to the analytic a posteriori. Unfortunately, neither of them mentioned any of the previous attempts, discussed above, so it is not surprising that both attempts reached very limited conclusions. First, Walter Block, in his brief overview of Kant's

fourfold epistemological framework [3], before relating it (like Cutrofello) to a field outside of philosophy, *starts* with a single paragraph on the analytic a posteriori, portraying it (very much along the lines suggested in “APK”) in terms of “the ways in which we come to learn language.”²¹ Unfortunately, having presented it as a valid classification, he makes no significant application of it in his essay.

Second and more significantly, Åsa Maria Wikforss devotes an entire article to assessing analytic aposteriority [8],²² concluding that it is *not* plausible to expect analyticity and aposteriority to be rendered consistent in the same way Kripke reconciled necessity and aposteriority. Appealing primarily to Tyler Burge (whose “externalism” never explicitly affirms the analytic a posteriori, but does seem to hint at it), Wikforss considers whether Burge’s approach could do for analyticity what Kripke did for necessity—i.e., make it a posteriori by locating necessity in objects (e.g., Hesperus and Phosphorus) rather than in descriptions. The problem, she claims, is that “the epistemic aspect is not so easily dismissed” in the case of analyticity.²³ This argument seems persuasive *if* we accept the same understanding of the basic terms that Kripke, Wikforss, and the analytic tradition in general adopt. My response to Wikforss, however, can be brief for precisely that reason: given the redefinition of Kant’s key terms that has become commonplace in the literature, the project of resurrecting the analytic a posteriori *is*, indeed, hopeless. But if we recover the original meanings Kant assigned to the key terms, as I suggested in “APK”, then Wikforss’ argument becomes a non sequitur: if Kripke’s insights *regarding the contingent a priori* (see note 23) amount to an affirmation of analytic aposteriority on *Kant’s* terms, then one who *accepts* those insights as valid cannot also deny the importance of the latter classification.

Although in this section I have only scratched the surface of its possible applications, the foregoing evidence should be sufficient to demonstrate the great importance of analytic aposteriority for contemporary philosophy. Were we to extend this study beyond philosophy proper, to areas of application such as philosophy of science, the relevance of the analytic a posteriori would prove to be even more relevant; for it would enable us to understand how Kant can claim at one and the same time that knowledge of the thing in itself is impossible (from the perspective of synthetic apriority) and yet to allow that scientists engaged in studying aspects of the world that transcend human observation (the level of the synthetic a posteriori) may in some sense be obtaining *knowledge* of the thing in itself (understood as analytic a posteriori).²⁴ I have elsewhere summarized this deep compatibility between synthetic apriority and analytic aposteriority in words that intentionally allude to Owen Barfield’s classic work [6]:

Classifying our *hypothetical beliefs* about the world [as analytic a posteriori] can do the crucial work of saving the *appearances*, both from being proudly mistaken for ultimate reality and from being discarded as *mere* appearances. The synthetic a priori class of knowledge occupied most of Kant’s attention; for he argued that all legitimate *transcendental* knowledge is of this type. This is why he said the question “How are synthetic judgments a priori possible?” is the central question of all Critical philosophy.

Kant fully recognized that Critical philosophy is a *propaedeutic* to metaphysics as such. What he did not recognize is that, in order to construct an actual system of metaphysics (even one that conforms to the educative principles laid down in the three *Critiques*), we must go beyond the synthetic a priori and immerse our inquiry in precisely the opposite ground. The extent to which twentieth-century philosophers recognized this need and have made genuine progress (often rejecting the letter of the Kantian law, yet if I am correct, following its spirit even more than Kant himself did) is the extent to which they have opened themselves up to that level of the human cognitive capacity that, in terms of Kant’s own framework, would have to be called *analytic a posteriori*.

References:

1. Aldrich V. C., “Analytic *A Posteriori* Propositions”, *Analysis* 28.6 (June 1968): 200-202.
2. Benardete J. A., “The Analytic a Posteriori and the Foundations of Metaphysics”, *The Journal of Philosophy* 55.12 (June 5, 1958): 503-514.
3. Block W., “Realism: Austrian Vs. Neoclassical Economics, Reply to Caplan”, *The Quarterly Journal of Austrian Economics* 6.3 (Fall 2003), pp.63-76.
4. Goldstick D., “Analytic *A Posteriori* Truth?”, *Philosophy and Phenomenological Research* 32.4 (June 1972): 531-534.
5. Kripke S., *Naming and Necessity* (Oxford: Basil Blackwell, 1980); hereafter, *NN*
6. Palmquist S., *The Tree of Philosophy: A course of introductory lectures for beginning students of philosophy*⁴ (Hong Kong: Philopsychy Press, 2000[1991]), p.77; cf. Owen Barfield, *Saving the Appearances: A Study in Idolatry* (London: Faber & Faber, 1957).
7. Quine W.V.O., “Two Dogmas of Empiricism”, *The Philosophical Review* 60 (1951): 20-43.
8. Wikforss Å. M., “An *A Posteriori* Conception of Analyticity?”, *Grazer Philosophische Studien* 66 (2003), pp.119-139.

Notes:

1. See especially the Introduction to Kant’s *Critique of Pure Reason*; hereafter *CPR*. All quotations cite Werner Pluhar’s translation (Indianapolis: Hackett, 1996), using the German (*Akademie Ausgabe*) pagination provided in the margins.
2. See, for example, Philip Kitcher’s various rejections of apriority in general and synthetic apriority in particular, as in “*A Priori* Knowledge”, *The Philosophical Review* 89 (1980): 3-23. For a response to such criticisms and a defense of the enduring meaningfulness of the label “*a priori* knowledge”, see Stephen Palmquist, “*A Priori* Knowledge in Perspective: (I) Mathematics, Method and Pure Intuition”, *The Review of Metaphysics* 41:1 (September 1987): 3-22.
3. See especially my article, “*A Priori* Knowledge in Perspective: (II) Naming, Necessity and the Analytic *A Posteriori*”, *The Review of Metaphysics* 41:2 (December 1987): 255-282 (hereafter, “*APK*”); my book, *Kant’s System of Perspectives: An architectonic interpretation of the Critical philosophy* (Lanham: University Press of America, 1993), pp.129-140 (hereafter *KSP*); and my chapter, “Philosophy of Religion after Kant and Kierkegaard”, in D.Z. Phillips and Timothy Tessin (eds.), *Kant and Kierkegaard on Religion* (London: Macmillan, 2000), pp.245-262; hereafter “*PRKK*”.
4. The rejection of this distinction by analytic philosophers is ironic, since the meaningfulness of the very name of this approach to philosophizing is called into question, if the basic opposition that gives rise to the term itself is considered meaningless. Yet this irony all too often escapes even the most prominent philosophers. I well recall the opening words of a series of introductory philosophy lectures delivered at Oxford University by Professor Sir Peter Strawson in the mid-1980s (paraphrased here from memory): “There are two types of philosophy in the world today: those that focus on solving philosophical problems through the analysis and clarification of language, and those that do not. In this course, we will deal only with the former type of philosophy.”
5. See note 2. For a collection of more recent essays illustrating the wide range of approaches to apriority, see Paul Boghossian and Christopher Peacocke (eds.), *New Essays on the A Priori* (Oxford: Oxford University Press, 2000).
6. Benardete raises the intriguing possibility, though only in passing, that Aristotle’s *Posterior Analytics* is, as its title already explicitly suggests, the science of the analytic a posteriori in just the sense Benardete is proposing (506). “The posterior analytic of color” is called “chromatics” (508,513), he goes on to suggest, whereas that of sound is called “sonics”.
7. Benardete, 508. He later adds: p.510: “The analytic a posteriori is founded on a principle expressly opposed to that of Hume, namely, that there are distinctions of reason which disclose empirical elements in reality which are distinguishable, and even different, from one another, though they are inherently incapable of being separated.”
8. Kant himself considers a very similar example in the Introduction to *CPR*, “All bodies are extended” (B11), and he agrees that it must be analytic. Interestingly, in the first edition he had used the example of “heaviness” (A8) to make the same point. Perhaps his change was prompted by a vague awareness that there is something odd about the necessity with which weight is included in our concept of a body—namely, we can know that containment relation only through experience, so the proposition must be a posteriori.
9. Even more surprising (and highly suggestive) is Benardete’s apparently essentialist conclusion: “Metaphysics is the posterior analytic of reality. Its object is to supply a posterior analytic of time and space, of motion and

- rest, of essence and existence, of being and non-being” (Benardete, 513), and in so doing, to provide “noetic” insight into the “eternal order” itself (514).
10. In what follows, I shall summarize the arguments presented in my article, “Knowledge and Experience—An Examination of the Four Reflective ‘Perspectives’ in Kant’s Critical Philosophy”, *Kant-Studien* 78:2 (1987), pp.170-200, subsequently revised and reprinted as *KSP*, chapter IV; see especially *KSP* 129-139. See also 366-369, for a summary of the application to Kripke, discussed in §4, below.
 11. In terms of Kant’s fourfold epistemological framework, the fault of speculative metaphysics can be explained as follows (*KSP* 130): “metaphysical reflection which has not been limited by a prior use of transcendental [i.e., genuine synthetic a priori] reflection will be patterned solely along the lines of a pseudo-transcendental mixture of empirical [i.e., synthetic a posteriori] and logical [i.e., analytic a priori] reflection: that is, it will attempt to produce synthetic a priori knowledge by conflating the logical perspective and it’s a priori aspect with the empirical perspective and its synthetic aspect.”
 12. As Kant himself warns, “practical cognition” provides a kind of “a priori cognition”, but *only* “from a practical point of view” (*CPR* xxi). As I argue in *KSP* 132-137, the change of standpoint from the first to the second *Critique* changes the context (from theory to practice, from science to morality) and thus changes the meanings of the key terms defining Kant’s epistemological framework.
 13. See *Critique of Judgment*, §59. For a similar passage relating to religious symbols, see *Religion within the Bounds of Bare Reason*, 6:64-65n.
 14. See note 15 for one major exception to this statement.
 15. Andrew Cutrofello, *Imagining Otherwise: Metapsychology and the analytic a posteriori* (Evanston, IL: Northwestern University Press, 1997); hereafter *IO*. He refers explicitly to the influence of “APK” in opening up the possibility of analytic aposteriority (pp.11-12; see also pp.25,162), but does not appear to be aware of the other texts where I defend its *compatibility* with Kant’s own philosophy.
 16. Cutrofello suggests, for example, that Hume’s own view of causality would, given Kant’s terminological framework, be classified as analytic a posteriori (*IO* 15).
 17. As one of many examples of Cutrofello’s attempt to psychoanalyze Kant, he claims that Kant shares with the Marquis de Sade an “underlying masochism” (*IO* 4)—though Sade’s masochism was, of course, quite explicit. Likewise, he interprets Kant’s claims regarding the necessity of the principles of pure understanding (especially the law of causality) not as philosophical arguments but, like “a dream-report” (62), “as clinical evidence—i.e., as an analysand’s statements about how he takes human experience to be structured.”
 18. “PRKK” also rejects Cutrofello’s undeveloped assertion that Hume may have been himself a supporter of the analytic a posteriori (see note 16); instead, I portray Hume and Hegel as following a trajectory that highlights the far more ordinary contrast between synthetic aposteriority and analytic apriority. “PRKK” also cites examples of analytic aposteriority in the philosophies of Kierkegaard (250-252), Otto (253-255) and Tillich (255-256).
 19. Cf. “PRKK” 250. Cutrofello’s negative (psycho)analysis of Kant’s position is unable to account for the fact that Kant was so explicit in his *affirmation* of faith in the ideas of reason, once we recognize that such faith is an expression of the very analytic aposteriority that Cutrofello accuses Kant of *denying*. I show, by contrast, that Kant’s affirmation of the analytic a posteriori shows up in *each Critique*, especially in the Dialectic and/or Doctrine of Method sections (“PRKK” 248-249), as well as in the fourth and final part of *Religion*, where Kant’s special definition of religion itself can be viewed as analytic a posteriori (249). The fact that Kant failed to give this classification a distinct *name* within his epistemological framework should not blind us to the fact that the *idea* of analytic aposteriority permeates every aspect of his philosophy.
 20. It is no accident that Heidegger’s *Kant and the Problem of Metaphysics* focuses on the faculty of *imagination* as the missing link to interpreting *CPR*. Kant’s repeated insistence that the imagination is a hidden faculty, whose depths we humans are never likely to plumb, is surely related to his refusal to acknowledge analytic aposteriority as a legitimate epistemological classification. Along these lines I would like to thank Guy Lown for his helpful comments on an earlier draft of this paper, especially his insistence on the open-endedness of the analytic a posteriori and on its relevance to twentieth-century phenomenology. As Guy aptly wrote in a recent email, “Hypothesis is the phenomenology of the fictional object.”
 21. *Ibid.*, pp.65-66. For further discussion of this classification, Block refers to 13 sources by six different economists: Hans-Hermann Hoppe, Murray N. Rothbard, Ludwig von Mises, G.A. Selgin, Glenn Fox, and Mario Rizzo. However, none of the works Block cites refer explicitly to the analytic a posteriori.
 22. Using the text at <http://people.su.se/~wikforss/Burge%20Berlin%204.pdf>, I cite the section numbers rather than the published pagination.
 23. *Ibid.*, §4.2. Significantly, Wikforss does not deal with Kripke’s theory of contingent apriority, where “APK” locates a type of analytic aposteriority.
 24. On the unknowability of the thing in itself, see *KSP* Appendix V. I have defended the compatibility of Kant and quantum mechanics in “Quantum Causality and Kantian Quarks”, forthcoming in *THEORIA: An International Journal for Theory, History and Foundations of Science*.



**The Adjustment of Identity:
Inquiries into Logic and Semantics of an Uncertain World**

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Abstract:

In this article I present some characteristics of logic and semantics of an uncertain world. I confront two-valued and fuzzy logic. I use Kafka's novel *Process* as an example, which is designed as an uncertain context with words which are rigid designators without rigid meaning. That produces an uncertain world of logical and semantical relations. In presentation of problems I introduce basic concepts of Frege's, Wittgenstein's, Tarsky's, Searle's, Quine's and Davidson's philosophy of language. I distinguish the logical and semantical identification of identity. Further, I make difference between reference and inference, or representation and identification as two components which are fundamental for the identification of identity. I ground this difference on the role of logical unification and granulation of predicates in the structure of thought and semantical unification and granulation of attributes in the structure of statements and their relation to ontology of context. Confronting the logical and semantical unification and granulation I find that the limits of logic are not also the limits of language. The semantical unification goes beyond the highest genre and below the lowest species. That enables the extra-logical, non-scientific, confessional, prophetic, artistic, and ordinary use of language.

Who was that? A friend? A good person? Somebody who was taking part? Somebody who wanted to help? Was he alone? Was it everyone? Would anyone help? Were there objections that had been forgotten? There must have been some. The logic cannot be refuted, but someone who wants to live will not resist it. Where was the judge he'd never seen? Where was the high court he had never reached? [...]

But the hands of one of the gentleman were laid on K.'s throat, while the other pushed the knife deep into his heart and twisted it there, twice.

Franz Kafka, *The Trial*

1. Introduction

What is the truth, for man to search for it so much, and what is man, to be searching for the truth so much?

Truth is a moving target in philosophy and science, but it is perhaps in art and literature that it moves at its fastest. The distance between us and the truth is also problematic: at times it is so near that our senses fail to recognize it; and sometimes it is so far that our mind only sees it in images itself produces. How, then, does the truth adjust itself to man, and how does man adjust himself to the truth? What is the relationship between identity and the truth?

Is that which we designate as the noun the “truth” and as the predicate “true” visual phenomenon or a mental representation of the visual phenomena, or is it only a linguistic property/predicate of some linguistic phenomena like that of the following propositions: (1) “The snow is white“, (2) “Bachelors are unmarried man“, (3) “Gold is a yellow colored metal“, (4) “The temperature is somewhere between the fifth and the sixth degree“, (5) “Salary is almost always paid between the 1st and the 8th in month“, (6) ‘Josef K.’ ” is the same person designated with the letter ‘K’ in Kafka’s novel *The Trial*“, or, (7) “The concept ‘identical twins’ designates two persons of the same sex who are genetically identical.”

Under which conditions are these propositions true? Is there some objective criterion applicable to all of these statements which would decide about their truth or the lack thereof? In what way does that which is claimed in these propositions adjust itself to what is and how it is outside of the proposition and, therefore, how does it adjust to that which is and the way it is in our senses and in our mind? Is there a procedure or some process that puts the language, world and thought into a *single relation* in a way that the identity claimed in these propositions becomes identical with the “identities” or “facts” or “states of affairs” outside the propositions, that is – in reality, or to those formed in mental state of affairs and in mental processes of our mind?

Apart from the *external adjustment* of thoughts and their expressions to the facts, is there some formula or a principle that would also enable the *internal adjustment* of the left and the right side of the identity sign or copula; something that enables the entire symbolical “reality” – one conceptual content – on the left side of the identity sign to correspond to the entire symbolical “reality” on the right side of the identity sign, as in propositions (8) “ $x+y=z$ ” and (9) “All blonds have same hair color”, so that everything is adjusted within the limits of quotation marks over the content of the propositions?

Through posing these questions we have suggested the possibility of differentiating the formulation of identities in those propositions that are dependent on ontology and of identities in propositions that are independent of ontology. If we wish to, we can name those ontology-dependent propositions the *uncertain propositions*, and these ontology-independent proposition *certain propositions*. This does not mean that we have substituted the traditional distinction between the analytic (experience independent) and synthetic (experience dependent) propositions with the new terms, it means only that we wish to open the question in a new way: why do rules for logical identity apply in “all possible worlds”, but not in the world of physical objects, particles in the gravitational fields, persons and their behaviors, their semantic and social history? Why is it that in these latter realities objects stand in different relations of “identity” (authenticity) other than that of this logical and theoretical? What is the relation of all these different “states of identity”?

In this text I wish to speak about the adjustment we accept to be *the truth* in interpretation (identification and re-identification) with particular regard to context (“obvious”, “logical” and “true”) as the *adjustment of the identity*, about modeling the identity of the persons, objects, facts, contexts, realities, cases, states of affairs. In particular I wish to speak about the logical and linguistic construction of identity (authenticity) of identity that ought to arise out of this adjustment.

Why is it necessary for identity to adjust, adapt and to be modeled in the perception of physical objects, in thoughts as well as in language? The truth is a daily being dependent on time, space and society; a being that appears and disappears, happens and verifies itself in our speech, in

interpersonal communication, in the interpretation of oneself and others. It is only here and in this way that it becomes objective [6]. The logic that we find in books and systems exists solely in this daily, language-arranged being; in communication and interpretation that re-arranges it, and only there can it be properly grasped and studied.

2. The Two-Fold Adjustment to the “Truth”

The question of *identity of identity* has been open to debate since Aristotle’s differentiation of synonymy, homonymy and paronymy [1, K. 1a1-15], i.e. the differentiation between the same (*auton*), the similar (*homoion*) and the equal (*ison*) [1, M.1021a10]; between that what we call substantial, qualitative and quantitative identity. Wherein does the logical and wherein does the semantical identity appertain to? It is quite possible that the science works on crossing this bidirectional road where identity is adjusted from periphery to the center, from perception to interpretation, and from interpretation towards things, from sense to reference [7] and whereon one relationship between the internal and the external is yet to be formed, a relationship which, under certain mental and space-time conditions, can be designated with the term “truth”.

I want to name this external adjustment “*semantic adjustment of meaning/re-ference*” or the adjustment of extension, and the internal adjustment I wish to name “*logical adjustment of sense/in-ference*” or the “adjustment of intension”. Words “direct to” or “point to” objects, words “relate to” objects, words “re-fer to” objects (things, properties, relations, events, processes, persons, human behaviors, words, sentences, thoughts). The way in which words relate to objects differs from the way in which propositions relate to objects.

The word/name “Aristotle” refers to the ancient Greek philosopher who was born in Stagira, to a Greek shipping magnate, to a computer antivirus programme, to a dog of one of the MTV’s popular singers, etc., while the set of words/predicative relation “ancient philosopher born in Stagira” refers only to Aristotle – the ancient philosopher born in Stagira. What is the rigid designator here: a proper name or the predicative part of the proposition? For names to be the rigid designators in all possible worlds, as Kripke claimed [14], they would have to belong to a single rigid semantical compress/context wherein either “predicative semantic sequence” (extension), or a complex semantical symbol from the other side of the equality sign, would always have to correspond to them. In my opinion, one should rather speak of the different ways of designating (referring to), sometimes even the same objects.

On the other hand, concepts involve, or are involved, they include each other, or are included in one another, they in-fer and inter-fere, they de-fine objects (things and concepts) in accordance with logical rules of subsumption and subordination. Concepts, in the whole of the conceptual content, differ from its predicative parts by the position they take and by the degree of logical generality they possess. Concepts can sometimes be identified with its predicative parts, and even substituted; other times, this is not possible.

Last of all, I wish to anticipate an additional point: a semantic adjustment of the identity of identity, or an adjustment of reference, is accomplished by the *semantic unification* of linguistic generalities belonging to the expressions out of which the proposition is built, as well as by the *semantic granulation* of attributive relations of a proposition through which it is possible to identify attributive states – the minimum and maximum of attributes – belonging to a certain object.

Contrarily, the logical adjustment of the identity of identity, or an adjustment of the intensity of logical generalities around the identity sign in a proposition, is accomplished by the *logical unification*/homologization of predicates, or by the inference of the values of logical variables, and by *logical granulation*. In other words: integration and distribution are two procedures, or two directions, or two ways of adjusting the identity of identity; they both operate in the structures of logic and language in the function of adjustment of thoughts and propositions with the objects they refer to.

We should now be precise: in the foundations of logical unification/homologization lies the logical/generic synonymy of the concepts of things. Logical unification is a procedure of homologization of logical generalities within the totality of a conceptual content which forms a

thought. Logical unification is directed by the highest genus, the one to which all degrees of logical generalities belong, and out of which the conceptual content is built, regardless of whether or not the conceptual content is divided into the subject and predicative parts. Logical granulation is an application of logical differentiations within the logical content. It is directed by the lowest placed class (species), i.e. the set of differences leading to it. Hence, a thought is a sequence of logical content with one limit in the highest genus and the other in the lowest class (species), regardless of whether they appear in that sequence or not. The minimum and maximum of logical generality of every thought is determined by these limits.

The analogy of the linguistic expressions lies in the foundation of semantic unification. Semantic unification is the homologization of linguistic generalities within a given complex linguistic expression that forms a proposition – from singular names to the abstract general expressions. Semantic unification is directed by the expression which, in a given ontological relation towards the object, includes the greatest number of analogical expressions. Semantic granulation is an application of the linguistic differentiation in attributive limitations within the description of an object with a finite number of expressions. A given expression of a given proposition is a sequence of analogical expressions which stand in attributive relation to one another, and whose upper limit is the object which the proposition refers to, and the bottom limit is a primitive (non-interpretative) attributive expression ascribed to all similar objects.

Thus far we differentiated *logical and the semantical unification* from the *logical and semantical granulation*; between the predicative and attributive relations, or, between the structures and the procedures belonging to language on the one hand, and those belonging to thought on the other. Attributes are the properties of things, and predicates are the characteristics of concepts. However, their interaction and dependence occurs in the context, within the limits of ontology of a context/a theory and its language, i.e. within the limits of the idioms of identity and quantification of one language and one culture (Quine), or perhaps within the limits of “my world” and “my language” (Wittgenstein), or does it have to occur in the limits of every language and every thought whose parts are articulated, i.e. they have sense and reference (Kripke)?

Amongst myriad others, there is one “holistic”, *Wittgenstein's principle*, which has dominated and still dominates a certain philosophy and science, and can be exposed in the following triptych: (1) that which one can precisely/clearly think, one can also precisely/clearly talk about, (2) whereof one cannot speak, thereof one must be silent, (3) whereof one can neither think nor talk, i.e. that which is unthinkable and unspeakable is only mirrored in language (*Tractatus*). With this, we have said that language and logic both extend within the same limits, that language does not go bellow or beyond the limits of logic nor does logic extend beyond or bellow the limits of language.

With this, too, we directly dismiss the possibility of uncertain ontology and uncertain language and uncertain thinking which could be identified as identical in any relation – either logical or spatial/temporal. But, how would the outer-logical, non-scientific, mythological, religious use of language be possible, how would that which Frege called *Dichtung* and the *Sprache des Lebens*, that which has sense (*Sinn*), but has no reference (*Bedeutung*), be possible?

3. Joseph K. in a Fuzzy World

Let us now take a look at an example of a thought content which can come into our consciousness by the force of the outer sensory stimulation of associative memory, and which can be formulated in a certain proposition. This proposition can be formed while we are sitting in, let us say, a *Free Speech Cyber Cafe* in Berkeley, drinking our coffee and looking at a person crossing the *Campus* lawn, heading towards the Moffitt-Doe library wearing a T-shirt with a big “K.” This can provoke a whole series of associations: (1): “Student with a *Calvin Klein* T-shirt”, (2) “T-shirt with the *Calvin Klein* logo”, (3) “*Calvin Klein* logo”, (4) “*Calvin Klein*”, (5) “Klein”, (6) “K.”, (7) “the character from the Kafka's novel *The Trial*”, (8) “member of the *Kappa* fraternity from the *International House* on the K/ Campus”. What is his name: Jusuf, Jasef, Josof, Josaf, Jesuf? How

many K's actually appear in Kafka's *The Trial*? How many people wear a Calvin Klein T-shirt, or how many members are there in the *Kappa* fraternity?

Which argument fulfills this "function": "object x has the property of K?" When precisely do we start thinking of the person, when of the thing, when of the complex symbol, and when of the simple sign that can refer to some other person? Which sequences of interpretation are involved, which are possible, and which are allowed? The semantic granulation of expressions, which in this case is related to physical stimuli and semantic history, produces one semantic net of relations (semantic compression) inside which the attributive relation functions. That relation can descend to the simple/primitive (non-interpretative) symbol, and ascend to the first logical form which the predicative relation begins to matter.

What kind of conceptual content can be created from the series of different representations provoked by a single sensory, external, physical stimulation of associative memory? What sort of mental and what sort of linguistic reactions correspond to this *uncertain physical input*? What is uncertain here: the *input* or the *output*, the stimulus, or the interpretation? Where does the interpretation take place? In the visual perception (retina), in the mental picture (somewhere in a parallel part of the brain), or in the semantic history of the sign (somewhere in the semantic zone of memory – in the amygdale, thalamus or hippocampus, which would be connected to the Broca's/linguistic zone in the brain), or in some experience of consciousness that would not be neurobiologically determined, as for example *Bewusstsein* or *Selbstbewusstsein* in Hegel's *Phenomenology of the Mind*?

To an understanding of a world which appears and disappears, to such an understanding of a truth which moves back and forth, and to such an understanding of an interpretation in which the truth, "truth" and the "truth about the "truth" "originate" in, corresponds a wholly different logic, the *fuzzy logic*, and a wholly different semantics, *the semantics of the uncertain world*. Inquiry into semantics of the uncertain world or into the theory of modeling of the meaning of words is prompted, encouraged by mathematicians, computational linguists, computational psychologists, by people who try to discover the semantics and logic of the world as imprecise, uncertain, unlikely, indefinable, and variable, and not just as it is already imagined. This, however, does not entail the unspeakable world.

Let us now move away from the mathematical and logical abstraction, and head towards the world of life and literature, and let us try to establish relations which, quite contrary to Rudolf Carnap's claim [2], occur to a man who is walking down the street where not a single motion is previously determined: birds fly uncertainly, cars move uncertainly, people walk uncertainly, the leaves of the trees in the Wilson's Avenue in Sarajevo fall uncertainly to the ground, the looks of the people uncertainly cross space and (its) objects / within it, the pedestrians and cars move together uncertainly from Miklošič's street across the Tromstovje Bridge in Ljubljana. The sounds around the river *Ljubljanca* are substituted in consciousness with the sounds coming from the river *Miljacka*. One external stimulus creates a net of related representations which consists of a minimal and maximal associative mental response to the received stimulus. One rigid stimulus never produces just one rigid and isolated representation in a subject's mental response.

Likewise, Josef K., the person representing the character in Kafka's novel *The Trial*, lives in an equally similar uncertain world without difference (the trial slowly turns into a conviction); in a world where concepts are uncertain, with no *in-fERENCE* (there is no clear logical relation of concepts), words are vague with no *re-FERENCE* (rigid designators do not have rigid meanings), the events are blurred, places are undefined and unadjusted to the events, the characters are also undefined and atypical, social relations are vague and uncertain. In the words of a modern logical and semantical theory of Lotfy. A. Zadeh [23], this person lives in a *fuzzy world*. The context of the novel is full of, not only linguistic variables and semantical generalities, but also, in Quinne's words, it is completely ontologically relativized and built on the basis of substitutive, rather than object-related interpretation of variables [17].

Josef K. himself, however, is a rigid type who searches conventional meanings, precise situations and precise relations; he demands a rigid or monotonic logic for a world in which he lives

(with two truth values: true or false, where a third does not exist (is non-existent) and the rigid moral he himself possesses. In contrast his trial is a *fuzzy* trial where nothing is certain, nothing is specified and nothing is given as a constant – the whole context is a variable. The laws by which Josef K. is being tried in his trial are in fact two pornographic books and one novel called “What Grete Suffered from her Husband Hans” [13]. Josef K. is arrested (exactly) on his 30th birthday, although he is not taken into custody and incarcerated; his trial has begun, but moves nowhere; he has an attorney defending him, but the attorney never leaves the bed; his legal hearings are not held during week days but on Sundays, and they do not happen in a court but in the attics of the barracks; the courts he visits are dark residential buildings on the periphery; he is the first procurator of a large bank, however, the investigator still asks him if he is a house-painter; his serious and sharp defense during his first and only hearing in an attic of a certain suburb is interrupted by pornographic sounds (screams of a woman) from the intercourse between a student and a laundress; the priest in the cathedral defeats his *two-valued logic* by turning each of his conclusions into an opposite syllogism.

Let us, at this point, make one *connectionist experiment* and highlight each letter K. that appears in *The Trial* with the felt-tip pen, and then let us put all the pages on one big surface so they are visible as one big jumbo-poster or one big screen. What one could then see is a DIAGRAM OF THE LETTER K WITH ONE DOT showing the whole book as a single-valued codebook whose “process” consists of a moving form, as a moving coded nonsense that has suddenly compressed. In this way it is possible for a new image to emerge, different from that which we get by simply listing the book from right to left: in a semantic compression created by the context of the novel. We can see that this stiff, rigid procurator from the bank who is represented by a single letter (K) and one dot (.) is actually a rambled and a scattered position in a well arranged nonsense.

This experiment suddenly shows the context as a rigid framework and the person becomes the *fuzzy* place, the uncertain topic, a moving target, scattered object, and a dot pointlessly spinning on the screen, a dot that stands nowhere in the mapping or in the equality of itself. This mapping into oneself is actually the very essence of the relation we call “identity” or “equality of the sets of elements” on both sides of the equality sign! Here, we discuss the object that cannot be compared with any other object in the context, nor can it be compared with itself. Only then it becomes visible how the *fuzzy* context was represented with more constants than the “object K.” in it. Solely thanks to this dot, placed under the lower cross stroke of the letter K., this object somehow still clings onto the context.

4. Josef K. in the *Tractatus*

One other Austrian, Ludwig Wittgenstein, in his opening sentence of *Tractatus logico-philosophicus* stated “The world is everything that is the case” [21], meaning that everything is just fine with our ordinary language: language which mirrors states of affairs, facts, cases and reality is just as precise as the world is: existing states of affairs and nonexistent states of affairs “p” and “not p”. Such a world and such a language have the same logical structure, a structure which enables the language to be the picture of reality (*die Wirklichkeit*), but not the picture of the world (*die Welt*). Wittgenstein did not allow space for the linguistic variables, nor do his terms “case”, “fact”, “state of affairs” point to an ontological relativity. On the contrary, the limits of my (rigid, without ontological variables) world, says Wittgenstein, are the limits of my (rigid, with no logical variables) language.

The world and the language cannot be in collision, simply because logic cannot be in collision with its application: if the world is rigid (facts, states of affairs, case) then the language is rigid too, words are rigid designators, regardless of the existing or nonexistent states of affairs! In fact, the world according to Wittgenstein can be only unspeakable but cannot be uncertain: if it is not a case or an elementary example of the world pictured in an elementary proposition. Reality (*die Wirklichkeit*) is that which is speakable and within it mirrors the unspeakable (*die Welt*).

Regardless the possible logic of the case might be, it is always in the service of positive sciences which create elementary tiny pictures of the world. There is not only one picture of the

world, there is no logical stratification, and there are only surface pictures which are created by the elementary propositions of positive sciences. There are only rigid descriptions, singular identity identifications of that-and-that, this-and-this, which appear like-this-and-this or so-and-so. The absence of logical unification (homologization of logical intensities which the abstract terms posses) has produced rigid and finite elementary semantic granulation (extension), one possible example of one logical relation, i.e. one symbolic or formal logical image of one material logical relation. In other words, the formal symbolic relation (proposition) is the picture of the existing or non-existing logical relation (fact, state of affair, reality)! This is why Wittgenstein was able to treat the general propositional form in two ways, both as a constant and as a variable (*Tractatus*).

Propositions in Kafka's *The Trial* do not reflect facts or states of affairs. On the contrary, negationless propositions describe, concern, and refer to nonexistent states of affairs. The fact that Josef K. *is not* guilty, the fact that the court *is not* a court (C= not C), the fact that the trial *is not* the trial (T= not T), the fact that the object to which the term refers to *is not* that object or is not such an object. Despite this, the propositions do not refer to the non-existing states of affairs, but rather take them *as* the existing states of affairs. Seen from the perspective of Wittgenstein's logic of the states of affairs as it is developed in *Tractatus*, the ontology of *The Trial* wherein Josef K. moves consists only of the nonexistent states of affairs, i.e. states of affairs expressed as "not p". There is no implication either, because nonexistent states of affairs can imply nothing but the nonexistent states of affairs.

Still, Josef K. makes material implications in his own logic even though the only things available to him are the non existing states of affairs. He concludes: if he is arrested then it *means* he is accused, if he is accused , it *means that* either he committed a crime or he is innocent, if he committed a crime, then it *means that* he should be convicted, if he is innocent then it *means* he should be freed. Guilty or innocent, there is no third option: the rigid implication in monotonic logic of normal process functions like that. The logic of the context in *The Trial* is twisted: if someone is formally declared as "arrested" and afterwards as "guilty," then it does *not mean* that someone is (conventionally) arrested and (conventionally) guilty! In a *fuzzy process*, he is only submitted to a psychological pressure: he is declared as "arrested" and "accused" and it is the only existing state of affairs in the novel which gradually proceeds into another existing state of affairs: into a conviction, without being arrested, without being incarcerated, without a hearing, without indictment, without defense, without the right to appeal.

Logic operates with the non existing states of affairs and treats them like facts of a negative auspice, which themselves belong to the possibility of logic. The non existing states of affairs mirror themselves in the propositions which posses a negation, propositions denying some existing state of affairs or some affirmation. However, semantics of non existing states of affairs are not the same as semantics or orthology of false speech: it shows/renderers nothing as a being, exactly as Plato defined it in the Sophist. Semantics of the non existing states of affairs is a semantics which does not begin with the linguistic variable that needs to be granulated or have its value set between the minimum and maximum, actually it is a semantics of an illusion. Josef K. goes through this semantics by trying to "build up " a system of rigid logic which, in this context has "fallen", instead of immersing himself into "his process" and "studying it seriously" in order to postpone the conviction as much as possible.

5. Josef K. in Wittgenstein's *Sprachspiele*

Josef K. starts to lose the trial before it had even begun, more precisely, in the moment when his name, appearing in the first sentence as the abbreviation "Josef K", is additionally abbreviated in the third sentence to the "K" only. The author of the novel does not allow even the slightest possibility for the character to be identified with the context. That is why the name "J-o-s-e-f" itself contains vowels (o, e) as phonetic variables, or as phonetically open or imprecise voices. The vowels in his name can, at any given moment turn into variables and bring into question the rigidity of his name, an option Kafka does not want. He even deprives K. of the possibility of uttering his name differently: e.g. as "Josaf", or "Jisaf" or "Jesof", or "Jusuf" or "Jesuf". When his name is

finally reduced to (a single letter) K., it loses even the nominal prospect of being something other than the rigid sign in the imprecise context. In the opening sentence of the novel the writer designates him as “Josef K.”, and immediately after, in the third sentence, he marks him merely with a letter “K.”. After that, he is referred to only with the letter ‘K.’ 1169 times. There are only several places where the sign K. is defined by the expressions: “mister”, “chief clerk”, “Josef”.

In his book *Logische Untersuchungen* Wittgenstein argued that the world and the language combined in one “language game” can lead to a semantic unification: language is a world for itself, each linguistic reaction – one “language game” – is the *function* of some *meaning* which words possess in their use or in their “grammar”. To learn the grammar of one word means to learn (all the ways) of its use [22]. He was so stricken with the meaning of the words (with the grammar, with the use of the words) that he even claimed that the words themselves observe us from the text! Is there anything more dangerous than the letter K. observing us from the novel, scattered in 1169 places? Taking Wittgenstein from *Logische Untersuchungen* as a witness, this could mean: 1169 meanings of the sign K.! It is an argument against the rigidity of meanings of rigid designators.

Franz K. completely reduced Josef K. to “K. in the trial”: all of his thoughts are 100% occupied with his trial, he is entirely devoted to performing the procedure that exists nowhere but in his memory, a memory that refers to the time before “the arrest”; to the procedure that should exist in a normal world; in his thoughts and in his expectations he sees the trial as a regular, normal procedure which has its course or its procedure in a legal state, while, in fact, it all begins when he wakes up in a *Workshop*, in one irregular, abnormal, non-procedural, informal process and trial.

Instead of living in the world of real meanings K. lives in a *workshop* (in his *trial*) where meanings of the words the “accused”, an investigating judge, a court, a trial, a lawyer exist only nominally, as terms which no longer have their full meanings (neither do the persons have full names!) which could be used in one way or the other, depending on the accused’s behavior. His trial is at the same time a diluted nominalistic, and a condensed psychological workshop (*psycho-word-shop*), an open synthetic function where the functions of the singular / individual terms do not exist. What actually rotates are only the psychological states of expectations, physical manipulations of his associative memory, instigations to wrong conclusions, and disappointments caused by the absence of real/genuine external events.

In such a *psycho semantic workshop* it is completely irrelevant if he is labeled as a “room-painter”, or “the first procurator of the bank”, what is relevant, however, is what kind of psychological and physical reaction this labeling causes inside him. The entire architecture of the context is nominal/istic (attendant, lower clerk, investigating judge, lawyer) and the meanings are *fuzzy* or uncertain because the reality they refer to is different, in other words, not an ordinary one. Likewise, his memory is also reduced to a “short term “memory; to cognizing faces and shapes which are present in the context. Despite the fact that he has no single recollection in the whole novel, he keeps searching for a procedure that belongs to a “long-term” memory.

K. would function brilliantly in a Bolle’s world which can be formed using the functions of the numbers 0 and 1, where 0 = “false” and 1 = “true”. In his procedural logic, he would subsequently create rigid descriptions of situations with just two symbols : 01, 10, 00, 11, 101, 110, 011, 010, 11111, 010101, 011011, 110110, 001, 10000010, 11110111, 11010101, 11111010, 10101111, 01110001, 11000111, 11001101, ...one endlessly arranged world of combinations of full and empty, one endless chain of sequences of “yes” and “no” with which one can count and that can be brought up into a convergence and divergence using different principles, even though, for instance, there is no difference in the content between the expressions 10101010 and 01010101. Nevertheless, he designates himself with the predicative relation of the words “I am the chief clerk in a large bank.”

On the other hand, the context of his (K.’s) civil lawsuit is irregular, informal, but at any given moment and at any given place, the actual, real, uniformed and rigid actors of the real process and the real trial can step into it. Kripke thought that the expressions are always rigid designators in every possible world [14]. However, one should add: if there is a rigid ontology, then there are also rigid meanings, words are thus (then) rigid designators. In a strange way the following is shown

here: rigid designators refer to both, non-existing and unspeakable states of affairs, but they do not refer to the uncertain ones.

Semantics cannot remain certain and rigid if the ontological status of the facts which words designate or refer to is being changed. Kripke could claim that the term “court” or “investigator” always refers to some object, any object, even imagined, non-existing object, but not to something like “a court which is different from the (real) court”, “investigator who is not the (real) investigator” or, “process which is not the (real) process”. This would then lead to a paradox of rigid designators.

In the previous section I have made an experiment that led us to the context of a *Workshop*. By doing so, with the help of Wittgenstein’s concept of the “language game” we have *fuzzyfied* (*fuzzyfication*) the linguistic constant K. and turned it into a linguistic variable, which was not the original intention of the father of *fuzzy* logic, Lotfy Asker Zadeh. However, since we have already found ourselves in the *Workshop*, we go one step further and make one more *fuzzyfication* in another way: animated fuzzyfication! If we would to repeat the same experiment with the letter K., only this time by using fluorescent felt-tip pen, and list through Kafka’s novel in the dark, in a way the authors of cartoons do it, letting paper sheets, placed between the forefinger and the thumb fall quickly, the rigid context would disappear from perception and the “letter K. with one dot” (K.) would create an animation of one movable target in literature that gets both, closer and further.

6. Josef K. in Searle’s *Chinese Room*

The question of relation between semantics and syntax in natural languages [3] as the question of sense and reference [7] and as the question of semantical and structural definition of the truth (truthful proposition) [20], is set out differently in cognitive science, artificial intelligence and computational linguistics. In the famous *Gedankenexperiment*, which he created in his text “Minds, Brains and Programs” entitled *Chinese Room*, John R. Searle makes an argument against the strong theory of artificial intelligence which claims that computers are intelligent physical systems which not only operate with symbols and perform structural procedures, but, are also capable of understanding the meaning and the semantics of symbolic sets [19].

Searle shows that these procedures can easily be performed by a man, while at the same time he does not necessarily understand the meaning of the alphabet, words, sentences, or the whole context that they together form. As an example he takes Searle who is completely unfamiliar with the Chinese alphabet, who knows not even one of the letters of the Chinese alphabet, closed in a room and given a set of the Chinese text, one set of Chinese letters together with the set of rules for a correlation of the subsequent set with the first one (the rules are given in English language, which Searle knows and which enables him to correlate one set of formal symbols with the other set of formal symbols); the third set of Chinese symbols together with the instructions in English which enable him to correlate the elements of this third set with the elements of the first two sets, and these rules give him directions on how to connect certain kinds of Chinese symbols to certain kinds of shapes as a response to a certain kind of shape given to him in the third set. Let the first set of elements be named “letter”, the second one “story”, third one “questions”, let the set of symbols that he connects as an answer to the third set of elements be named “answers to the questions”, and may the set of rules given to him in English be named “programme”. Searle claims that now, by following the rules he understands since they are in English, the language he knows, he will be able to put together the elements taken from different sets of the Chinese characters text and piece a story in a Chinese language thou he would not understand it. Through connecting the elements and their correlating, he produces an answer (*output*) out of what is given to him in the room (*input*) by manipulating non-interpreted symbols. He simply behaves as a computer running computational operations with formally specified elements. Therefore, Searle concludes, (in order) to function in one context it does not imply understanding it, just like the computer and the programme are functioning, although they do not comprehend [19].

Let us this time confine/close Josef K. or simply K. instead of Searle himself in Searle’s *Chinese Room*. He does not know the semantics of the world he lives in; in fact, he is not familiar

with the *fuzzy* semantics because his semantics is rigid, semantics of the rigid designators, semantics of every possible world (Kripke), but not the semantics of every possible reality. However, the problem is even bigger in so far as, unlike Searle in Chinese room, K. does not receive neither precise procedural instructions nor the rules for connecting or correlating the elements of events in the context of *The Trial*. He actually has no directions whatsoever, and he is asked nothing else but to get carried away in his position (that he is the guilty one) and this is precisely what he is incapable of doing because he is constantly carried away that he is the innocent one. Josef K. is sharp (crisp) upon every contact, his claims are sharp his offsets are harsh, his logic is, in terms of Lotfy A. Zadeh the “*crisp logic*”. From the beginning to the end of the novel he functions but does not understand anything, his functioning does not gradually evolve into an understanding in the way his trial gradually turns into conviction. From the viewpoint of the semantics of context, his process has neither sense nor reference. The events in his process are unrelated and do not follow (by) any rule. His trial has no truth value, because the words have no rigid meaning. His case could possibly be represented by function of belonging in one *fuzzy* set, yet his problem would not be solved by it. This means that such a context has no truth value: “absolutely true” or “absolutely false”. What is missing for the context to have meaning is *identity of identity* (authenticity) or identity of the words with their meanings. The process is nominalistic, the meanings of the terms used are uncertain, the words are not rigid designators, for the most part there is no reference (object, state of affair, fact). Josef K. manipulates with the terms, words: the guilt, accused, questioning, defense, lawyer, court, clerk..., but he is not able to put together the rigid context, because the order of events is uncertain, and because the meanings are uncertain; he cannot interpret nor identify the world that exists behind these terms. In other words: the context of the novel *The Trial* is given in formal implications, but there are no material implications, consequently there is no possibility for recursive definition. The semantics of an uncertain world would actually be, in Aristotle’s terms, homonymous identification of identity which is the basis for homonymous predication – things have a common name, but yet a different notion of essence designated with it (with a name). This “concept of essence marked with a name” (*kata tynoma logos tes ousias*) plays an important role in determining the meaning of words and sentences in Aristotle’s logic and semantics: definitions created on homonymy are based on attributive heterological relation and not on the predicative homologization, they are logically unclear because they are based on coincidental relation of attributes with the substratum. The definitions of homonymous things are not the same (*auton*) but attributive (*idion*) [1, K. 1a1-15].

Josef K. is not able to interpret the symbols that surround him in the context because they are set up only as formal elements of one context, as common terms without firm meaning or the essence they designate, as words which have no reference or do not have a convention based ontology. Besides this, he has no precise instructions how to use these formal elements. He has no single direction on paper. He does not even have the invitation to go to court. He has no pile of documentation about his annual trial. He is being invited to questioning orally, over the telephone. His trial does not officially exist, but everybody knows about it. He interprets everything wrongly because he does it from his internal mental set, from his inner linguistic room, from his rational cage, wherein the *homunculus* Josef K. acts, which holds everything certain (precise), regulated, procedurally memorized, but to what nothing in the world corresponds, neither semantically nor structurally.

7. Identification of Identity of Identity

Logicity of one thought depends on the technique of unification and granulation of the conceptual content that constitutes one thought, and the meaning of one proposition depends on the technique of unification and granulation of parts of the propositions/expressions: sentencehood of a sentence depends on the semantic use and on the structural arrangement of its parts. What controls this arrangement? The arrangement of logical forms is controlled by the logical apparatus of quantification, identification, and generalization. How does one stand with the semantical arrangement? How is sentencehood accomplished? What is that which carries out the

“homologization” of linguistic forms (unification of designators) which is necessary to accomplish the identity of identity in language?

Analytic philosophy has opened a programme of demands so that the *sentencehood of sentences* of the natural language could be submitted to the *logicality of a logical form* of the proposition in that language. The demand for logic to be the grammar of a language – originally posed by Frege and then widely accepted and nurtured all the way up to the dreams of canonical notation, to mechanical translation of idioms of thoughts into the idioms of expressions [3] – actually tells us that the field of language is the field of contingency, and that the field of the logical is the field of necessity. To reckon with the contingency would mean breaking up with the terror of language over thought [9].

Let us examine these demands from the point of their essence. What does it mean to speak about the logicality of a thought? The logical structure of a thought, according to Frege [8], depends on the relationship between the parts of a conceptual content (*Gedankengefüge*), on the connectives (logical constants, logical operations) that stand between them, on the level of logical generality that one conceptual content possesses, on the quantifier that relates to the predicative part that is made out of conceptual words (*Begriffswörter*). The *homologization of predicates* is the basic law which should lead to the logicality of thought or to logical identity of identity in logic: to equal arrangement of logical generality of parts of the conceptual content that stand on the left and the on right side of the identity sign. This is accomplished only if all the parts of the conceptual content belong to the same genre, from the highest to the lowest; from the highest genre to the lowest species (subordination). Only then is there a cognitive synonymy and definition and “substantial identity” [12]. Only then do things have (a) common (and not the same) name and (the) same (and not a common) concept of the essence designated with a name (Aristotle).

We could also pose a question about the languageness (languagehood) of a natural language, like the language of Papuans. How much do the elements and structures of their articulated communication possess the abstractness of language and how many of them are mimetic and onomatopoeic in their character? From which relation should one derive answers to these questions, from comparison with our language, from comparison with similar languages, or from the investigations of the use of that very language itself, thoughts it expresses and reality it refers to? Hence, we have: the meaning of one sentence, the meaning of one set of the sentences, sentencehood of one sentence and sentencehood of one set of the sentences. But in the end not / but in the end we do not have: languagehood of one language – whereof it all depends?

We should not forget one other Wittgenstein’s claim from *Logische Untersuchungen*: “*Einen Satz verstehen, heißt, eine Sprache verstehen. Eine Sprache verstehen, heißt, eine Technik beherrschen*” [22]. For Wittgenstein it meant: to know the rules of the use of words and sentences in one language game. But, we can observe this from the viewpoint of that what is logical in a language game and say: technique that should be mastered is the technique of unification and granulation of language expressions and logical forms.

Is it not the same as speaking about the musicality of one music piece, about how its parts are arranged, do they hold together with one law of tone array and do they create *one* tonal whole, be it harmonic or disharmonic? Isn’t the tonic unification, a symphony that replaces the synonymy at stake here? Isn’t it the same as to speak about the artistry of one drawing or painting, about the photographness of photograph? But, we speak of the photogenicity of a person: some person is photogenic, though not pretty. We make a difference here between the content of photograph, a person or some object, and the very form of photograph, its structure, relation of photo-elements. A person is photogenic, a photograph is photographic.

Some artistic painting is not as photographic as a photograph, but the set/the whole of color components, lines, surface, perspectives, voids, objects and their formation is a synthesis giving the painting the characteristic of splendid work of art, which, for example, truly represents one scene in reality, though it does not show something beautiful (Rembrandt’s painting “*The Anatomy Lesson of Dr Nicolaes Tulp*”).

Thus far, we have spoken of the logicity of propositions, even though it is spoken about the logicity of some acts too, some events, some actions, and some process. But, what is logic of propositions principally? I want to say something anarchical here: the logic of propositions is not concerned with the expressions of propositions, but with the thought it expresses, it is actually to be found in the relation between the parts of the conceptual content and this *relation* is that which enables that *one* part, assembled of a large number of expressions, expresses one thought for which we say is logical or illogical proposition.

Thoughts too have their parts. In logic, in order to be logical or illogical one thought always has to be assembled out of two parts at least: *if* part and *then* part. In logic, thoughts are always compounded of the premise and conclusion, antecedent and consequent. Though, there is one more thing to anticipate here: logicity of logic of some expression enables the identity of parts out of which the thought expressed in proposition is composed of. Belonging of all the predicates of one subject into the same genus, hence: substantial identity or cognitive synonymy. To what then does the predicate “true “ (proposition) refers to in propositions: to language expression or to thought expressed? Can this possession over the predicate true be discovered out of the very proposition itself, or do we need certain proposition about this proposition wherein we would say that it is true or false?

8. Recursive Adjustment of Identity

Cognitive synonymy is a recursive logical function composed of that which is *common* and that which is *same*. Form is that which is common and content is that which is same. Word / expression is that which is common and concept is that which is same. It is the model of logical and linguistic equivalence, model of equivalence of formal and material implication and model of equivalence of the world and reality. This is why Davidson held blindly to Tarski’s convention T, i.e. one-referring, mutual biconditional in trying to give formally satisfying and materially adequate theory of truth for one natural language or natural languages in general, even though Tarski used it for formalized languages only (the language of the calculus of classes) [20].

In logic, however, the paraphrase is not the same what the recursive definition is: paraphrase repeats the content, not the form, it transforms one and the same thought or one and the same logic of proposition into another expression, as in Tarski’s example of proposition inclusion (x_I) and (x_{III}) and the negation of their inclusion: “ Ix_Ix_{III} ” and “ Nix_Ix_{III} ” formulated in the paraphrase $((((ng(inx_I)x_{III}))$ by which one logical relation of elements appears in two different language ways. Recursive definition, however, repeats both – the content and the form – with the exception that it puts predicate “is true proposition” (Frege’s “fact that _____”, or Wittgenstein’s expression “case that _____”) in metalanguage, in a predicative part, as is the case in the following example: “Proposition ‘Snow is white’ is true if and only if the snow is white”.

Let us now consider this from the viewpoint of difference that I want to introduce in this text, the difference between the logical unification which functions within the content (of thoughts, conceptual content) and semantical unification which functions in the expression of one thought, i.e. from the viewpoint of difference between (the) predicative and attributive relation through which the homologization of the content and of expressions on the both sides of the identity sign is accomplished by. It has already been mentioned that in the sense of logical reference the identification of identity of identity or the homologization of content of thought is based on *predicative/cognitive synonymy* by which the sense/thought/conceptual content is adjusted/arranged, and it has also been mentioned that in the semantical sense the identification of identity of identity or the semantic unification occurs/is based on the *homonymic predication/attribution* by which the meaning/reference/signified is adjusted.

I would like to supplement this relation of inference and reference now with the next characterization: the semantical unification of expressions goes above the highest genre/genus to which the logical unification/homologization of predicates reaches, and semantical granulation goes below the lowest species to which the logical granulation reaches. This means: *the semantical maximum and the semantical minimum do not coincide with the logical maximum and the logical*

minimum. This enables the language to function in the extra-logical, irrational, metaphysical, mythological, poetic, confessional and prophetic constructions.

For the moment let try to expand Tarski's T convention to show this duality of logical and semantical characterizations: (1) "Proposition 'Snow is a kind of precipitation' is true if and only if the snow is a kind of precipitation", (2) "Proposition 'Snow is white' is true if and only if the snow is white". In the proposition (1) "A=A" under the condition that "A=A", and in (2) "A=B" under the condition that "A=B". What's the difference? In (1) the relation is substantial, the subject is identical to itself, and in (2) the subject is partially or qualitatively identical only to one of its attributes.

The relation in (1) is generic and can be generically granulated: "Snow is a kind of precipitation consisting from pieces of tiny crystals of ice". In (2) generic granulation is not possible because it involves the attribute (real property, description) and not the predicate (logical characteristic, definition). Is the identity of these propositions adjusted in the same way or do we speak about the same degree of identities in both cases?

The unification of variables, logical and linguistic, should ensure the inner identity of identity (sense), and this implies substantial appropriation of the set WORLD {S} and the set LANGUAGE {L}. In the recursive adjustment of the identity of identity cognitive synonymy and linguistic synonymy contribute to connecting *the adjustment of inference* (logical identity, oneness, identity of conceptual content, substantial identity) *and to adjustment of reference* (linguistic identity, qualitative and quantitative identity, equality, similarity, identity of the meanings of words and the objects designated/signified, the sign and the signifier). Only in this way, by unifying the forces which have these two components, identification and representation, the adjustment and fitting to the "truth" aimed/targetted is possible: identity (identities, equalities and similarities) of the elements in the ontological structure of the being.

9. Conclusion

The central theme of this text could have been a consideration of Carnap's claim "To be is to be an element of the system" [2], or Quine's claim "To be is to be the value of variable" [18], or Wittgenstein's claim: "To understand one proposition is to understand one language...". This consideration, however, has immediately turned itself into a question of constant and variable, of rigidity of the constant and the *fuzzy* attributes of a variable. Can Quine's diagram be read as "To be an (entity) is to be the *fuzzy* value of variable?". Analytic philosophy became famous for putting on and taking off the quotation marks in trivial propositions "Tarzan loves Jane" and "Jane loves Tarzan" (TIJ & JIT), for searching the jungle of relations and classes of relations in order to find a place for a truth-valued predicate between the singular and general terms (predicates).

However, in the proposition "Tarzan is the king of animals", proper name is a rigid designator, while the predicative part is the logical structure of predicates of different generality. The concept "king of animals" is granulated into subordinated concepts: "king of terrestrial animals", "king of sea animals", "king of bipod animals", "king of four-footed animals", "king of reptiles"up until the last species and subspecies of beings included into the scope of the concept of animal. In the proposition "Tarzan is the king of jungle" the concept "king of animals" is implicitly given in the concept "king of jungle", included and unified in the logical space as logical generality of a certain rang. Let us observe the proposition "The snow is white": the noun "snow" is rigid designator, while it is impossible to granulate the predicative part within the same species, i.e. we would have to find a comparison (analogy) in the different species: as some other white object ("as milk").

The predicative part of the proposition or the context of the logical variable is *the compress of logical generalities* that can be granulated by going top-down like Plato claimed long ago (*Sophist*) that one should start from the highest fitted genre and descend by dividing each form into two forms up until the last species which can no longer be divided. Aristotle named this last species '*eshaton eidos*' or '*eshata ousia*', and Porphyry as *eidikotaton eidos*.

Context has different levels of generalities: logical, ontological and semantical. *Logical granulation* of the concept “animal” and semantic granulation of the linguistic variable “animal” do not correspond, because in the first case what acts is *the predicative homologization* whose sphere goes only from highest genre to the last or the lowest fitted species, while what acts in the second case is the *attributive difference* which goes above the highest genre and lower than the last species to the granulation of accidental property (*symbebekos*).

Logical unification of variables, one procedure in monotonic logic that ensures identity and synonymic or cognitive belonging of all the relates on the two sides of the identity sign with the procedure of generic homologization or substantial integration of predicates (sufficient for the logical concept of truth that stands on the generic line of predicative parts of the content), has opposed to itself the *semantic granulation of variables* (so called *Computing with Words*), one procedure which came to term specially in *fuzzy logic*, which serves for adjusting the identity in the uncertain situations or contexts and one which stands on the attributive differentiation or on distribution of meanings of the expression parts for content that goes between the minimal and maximal belonging to one subject.

In the end: Who killed Josef K.? One? Someone? Everyone? The same context that killed K.? The context that killed “the first procurator of one large bank”? Fuzzy logic of an uncertain world? Context that killed “the house painter”? His rigid logic? His reactions caused by the uncertain context of the *Workshop* where he woke up in on his 30th birthday? The semantics of the nonexistent states of affairs in which he woke up, and for which he was trying to find an adequate logic? Logic or semantics or an invisible ontology? Visual or intellectual culture, visual or intellectual mentality, visual or intellectual states of affairs and processes? Nonexistent states of affairs, non-being that appears as the other of being? Repressive context of physical stimulations that started his perception and created psycho-nominal(istic) net of associations?

Philosophy and art, science and religion, have to seek answers to those kinds of questions in the ontology of an uncertain world, a world which has its own logic and semantics in the same way the ontology of the certain world, which exists only in transcendental-mathematical or theoretical constructions enabled by the rigid logic and rigid semantics has them.

Translation: Tijana Okić

References

1. *Aristotelis Opera*, Academia Regia Borussica (ed.), Ex Recensione Immanuelis Bekkeri. Volumen Primum. Editio Altera, Berolini, 1960.
2. Carnap, R., *Meaning and Necessity*, Chicago: The University of Chicago Press, 1956.
3. Davidson, D., “Semantics for Natural Languages” [in:] Davidson, Donald, *Inquiries Into Truth and Interpretation*, Oxford: Oxford University Press, 2001.
4. Davidson, D., “Reality Without Reference” [in:] Davidson, Donald, *Inquiries Into Truth and Interpretation*, Oxford: Oxford University Press, 2001.
5. Davidson, D., “On the Very Idea of a Conceptual Scheme” [in:] Davidson, Donald, *Inquiries Into Truth and Interpretation*, Oxford: Oxford University Press, 2001.
6. Davidson, D., *Subjective, Intersubjective, Objective*, Oxford: Clarendon Press, 2001a.
7. Frege, G., “Über Sinn und Bedeutung” [in:] Frege, Gottlob, *Funktion, Begriff, Bedeutung*, Göttingen: Vandenhoeck & Ruprecht, 1962.
8. Frege, G., “Gedankengefüge” [in:] Frege, Gottlob, *Logische Untersuchungen*, Göttingen: Vandenhoeck & Ruprecht, 1960.
9. Frege, G., *Begriffsschrift und andere Aufsätze*. Zweite Auflage. Mit E. Husserls und H. Scholz Anmerkungen, Herausgegeben von Ignacio Angelelli, Hildesheim: Georg Olms, 1964.
10. Frege, G., “Logik” [in:] Frege, Gottlob, *Logik und Sprachphilosophie*. Hamburg: Felix Meiner Verlag, 1986.

11. Ibrulj, N., *Filozofija logike*, Sarajevo: Sarajevo Publishing, 1999.
12. Ibrulj, N., "Holizam identiteta", *Pismo*, Sarajevo, 2004, pp.
13. Kafka, F., *The Trial*, Vintage Books, 2005.
14. Kripke, S., *Naming and Necessity*, Oxford: Clarendon Press, 1972.
15. Quine, W. V. O., *Word and Object*, Cambridge, Massachusetts and London: MIT Press, 1960.
16. Quine, W. V. O., *From a Logical Point of View*, New York: Harper, 1961.
17. Quine, W. V. O., *Philosophy of Logic*, New York: Prentice-Hall, 1970.
18. Quine, W. V. O., *From a Logical Point of View*, Second Edition, Cambridge, Massachusetts and London: Harvard University Press, 1980.
19. Searle, J. R., "Minds, Brains, and Programs" [in:] Haugeland, John (ed.), *Mind Design*, Cambridge, Massachusetts and London: MIT Press, 1980.
20. Tarski, A., "The Concept of Truth in Formalized Languages" [in:] Tarski, Alfred, *Logic, Semantics, Metamathematics*. London: Oxford University Press, 1956.
21. Wittgenstein, L., "Tractatus logico-philosophicus" [in:] Wittgenstein, Ludwig, *Schriften*, Frankfurt am Main: Suhrkamp Verlag, 1960.
22. Wittgenstein, L., "Logische Untersuchungen" [in:] Wittgenstein, Ludwig, *Schriften*, Frankfurt am Main: Suhrkamp Verlag, 1960.
23. Zadeh, L. A., *Fuzzy Sets and their Application to Cognitive and Decision Processes*. New York: Academic Press, 1976.

Is Identity a Logical Constant and Are There Accidental Identities?¹

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Abstract:

In the paper I discuss whether identity is an extralogical problem. Then I show that identity in Kripke's meaning when any identity implies necessary identity may be called unconditional identity and it is a special case of conditional. As a result, we obtain a uniform treatment of $=$, independent of the view whether it is a logical constant or not.

Propositional connectives and quantifiers are logical constants without any doubt. On the other hand, we speak about first-order logic with or without identity. Even this way of speaking suggests that identity has a special status to some extent. In fact, the status of identity is controversial. Wittgenstein says ([6], 5.5303):

Roughly speaking, to say of *two* things that they are identical is nonsense, and to say of *one* thing that it is identical with itself is to say nothing at all.

According to Wittgenstein, identity is not a relation. This view rises an important point: does identity can hold between objects, which are numerically different, for example, two occurrences of 'e' in the word 'different'? Tarski's view of identity was radically opposite to that of Wittgenstein. The difference is well illustrated by the following quotation ([4], p. 49):

Among the logical concepts not belonging to sentential calculus, the concept of IDENTITY, or, of EQUALITY, is perhaps the one which has the greatest importance.

Wittgenstein's and Tarski's statements about identity can be rephrased without essential changes by replacing 'identity as a relation' by 'identity as a predicate' (I prefer the second way of speaking).

Formally speaking, identity is introduced into first-order logic by the axioms (I omit quantifiers in the front of formulas)

(A1) $x = x$;

(A2) $x = y \Rightarrow y = x$;

(A3) $x = y \wedge y = z \Rightarrow x = z$,

together with the rule of replacement (for simplicity, I restrict it to monadic predicates, but it is obvious how to generalize this rule from arbitrary formulas)

(RR) if $(x = y)$, then $P(x) \Rightarrow P(y)$.

Thus, first-order logic with identity (**FOLI**) is determined by propositional calculus (**PC**, more precisely by its codification via axioms or rules of inference), pure (the meaning of 'pure' in this context will be explained below). first-order logic without identity (**PFOL**, codified by axiomatization or rules of inference) and the set $\{(A1)–(A3), (RR)\}$.

As it is well known, the identity predicate is not definable in first-order logic. The situation changes in second-order logic via the Leibniz rule:

(LR) $(x = y) \Leftrightarrow \forall P(Px \Leftrightarrow Py)$,

which says that identical objects have the same properties. In fact, the implication

$$(1) (x = y) \Rightarrow \forall P(Px \Leftrightarrow Py)$$

suffices for defining identity. The reverse implication

$$(2) \forall P(Px \Leftrightarrow Py) \Rightarrow (x = y)$$

expresses the famous *principium identitatis indiscernibilium* (the principle of identity of indiscernibles). Hence, (LR) the conjunction of (1) (the principle of indiscernability of identicals) and *principium identitatis indiscernibilium*.

Although formal properties of identity are (or seem be) clear, the concept of identity provides several problems for logicians and philosophers. One of them is captured by the already mentioned question ‘Is identity a logical constants?’. The arguments for the affirmative answer point out that fundamental metalogical results (semantic completeness, compactness, undecidability, the Löwenheim-Skolem theorem, the Lindström theorem) are valid for **FOLI**. In particular, the last results seems important because it provided a characterization of first-order logic as contrasted with higher-order logic. Consequently, the the Lindström theorem determines the borderline between ‘being the logical’ and ‘being the extralogical’, that is, the first-order thesis (see [7] for details and a discussion). Speaking metalogically, all theorems with the identity-predicate derivable in **FOLI** are universally valid. Speaking more philosophically, these theorems are necessary in the strongest sense, because logic represents an uncontroversial kind of necessity. Tarski (see [5]) argued that identity is a logical notion because it is invariant under all transformation of a domain into itself.

However, there are some problems with considering identity as a purely logical item. Having identity, we can define numerical quantifiers of the type ‘there are n objects’, where n is an arbitrary natural number. Consequently, we can characterize finite domains, although first-order logic is too weak in order to define the concept of finiteness. Now, if we add the sentence ‘there are n objects’ to first-order logic, its theorems are valid not universally, but in domains that have exactly n elements. Hence, it seems that identity brings some extralogical content to pure logic, contrary to the view (it can be expressed by a suitable metalogical theorem) that logic does not distinguish any extralogical content. Perhaps this is a very reason that the label ‘the identity-predicate’ is used, although logicians simultaneously remark that this is a very special predicate. Anyway, a qualification of identity as logical or extra-logical is conventional to some extent.

Other reason to see identity as an extralogical problem stems from so-called inflation and deflation theorems (see [3]), both closely related to the definability of finite domains in **FOLI**. The former says that if a formula, let say A , is satisfied in a non-empty domain $\mathbf{D}$ with n elements, it is also satisfied in any domain $\mathbf{D}'$ with at least n elements. The deflation theorem asserts that if A is satisfied in $\mathbf{D}$, it is also satisfied in any $\mathbf{D}'$ with at most n elements. Although these theorems hold for **PFOL**, they fail for **FOLI**. The formula $\forall xy(x = y)$ provides a counterexample to the inflation theorem, because it is true in the one-element domain and no other, but the formula $\exists xy(x \neq y)$ is false in the domain with one elements. On the other hand, both theorems hold in **PFOL**. This is a reason for applying the adjective ‘pure’ to first-order logic without identity. On the other hand, if we look at **PC** and **PFOL**, we can note some metalogical difference between both systems. In particular, **PC** is decidable, but **PFOL** has no decision procedure. Furthermore, **PC** is Post-complete, but **PFOL** (with numerical quantifiers) lacks this property. This shows that the concept denoted by the phrase ‘being the logical’ has a different strenght in particular subsystems of **PFOL**.

One additional problem requires a clarification. According to early Frege (see [1]) and Wittgenstein (see [6]), identity operates on signs. The view that the formula $x = y$ concerns objects became standard in contemporary logic. However, the notation used in (A1) – (A3) (as well as in other quoted formulas) is ambiguous to some extent. In fact, under the objectual treatment of identity, we should formulate (RR) as

$$(RR') \text{ if } (d(x) = d(y)), \text{ then } P(d(x)) \Rightarrow P(x/y).$$

This formula means: if the object denoted by the letter x (the denotation of x) is identical with the object denoted by the letter y (denotation of y), then if the denotation of x has a property P , the letter x can be replaced by the letter y . Note that the antecedent of (RR') concerns object, but its

consequent deals with objects and signs. A suitable rephrasing of other listed formulas is straightforward. The proposed reading of the replacement rule underlines its semantic character. We should think about identity as determined by an interpretation of terms in models; denotations depend on valuation functions ascribing objects to terms (individual constants and individual variables; I omit valuations of predicated letters). Note that (A1) is the only axiom, which is an unconditional formula contrary to (A2), (A3) and (RR) (or (RR')). I assume the objectual reading of identity in what follows without using symbolism employed in (RR').

Kripke (see [2]) presents an argument intended to show that there are no accidental identities or that every identity is necessary. This view is supported by the following reasoning. Assume (RR) and (A1) in the form (I insert quantifiers, because this proposition plays a relevant role in the argument; the box expresses necessity):

(A1') $\forall x \Box(x = x)$ (every object is necessarily self-identical).

Now, interpret P as the property 'necessarily identical with'. (RR) gives

(3) $\forall xy(x = y) \Rightarrow (\Box(x = x) \Rightarrow \Box(x = y))$.

Since $\Box(x = x)$ is universally valid, it can be omitted. Thus, we obtain

(4) $\forall xy(x = y) \Rightarrow \Box(x = y)$,

that is, the conclusion that if two objects are identical, they are necessarily identical. However, this result seems non-intuitive, because the identity of London and the capital of UK looks as accidental.

How convincing is this reasoning? First of all, let us change it by using the provability operator $\vdash$. Since (A1) – (A3) are logical axioms (I assume here that identity is a logical constant), we can add $\vdash$. As far as the matter concerns (RR) (translated into a formula), we obtain (quantifiers added):

(RR'') $\forall xy(\vdash((x = y) \Rightarrow (P(x) \Rightarrow P(y))))$.

Since the provability operator is monotonic, (RR'') entails

(5) $\forall xy(\vdash(x = y) \Rightarrow \vdash(P(x) \Rightarrow P(y)))$.

Two things are to be observed. Firstly, the antecedent inside (5) has the sign $\vdash$. Is it possible to skip this element in (5). It would be at odds with the practice of using identity in inferences. For instance, mathematicians derive conclusions about properties of identical objects, assuming that its identity is provable in mathematical theories. Secondly, we cannot interpret P as expressing provability. Now, if provability is understood as a kind of necessity, Kripke's argument cannot be repeated. We can only obtain:

(6) $\forall xy(\Box(x = y) \Rightarrow \Box(P(x) \Rightarrow P(y)))$.

Inserting or omitting the formula $\Box(x = x)$ is completely pointless in this reasoning. Let us strengthen (5) to the formula

(7) $\forall xy(\vdash(x = y) \Leftrightarrow \vdash(P(x) \Leftrightarrow P(y)))$.

We can think about (7) as a scheme capturing the first-order version of the Leibniz rule. Disregarding whether the provability of the right part of (7) is realistic or not, this equivalence leads to

(8) $\forall xy(\Box(x = y) \Rightarrow \Box(x = y))$,

which is not very exciting, because it asserts that if an identity is necessary, it is necessary.

I will not discuss Kripke's solution of the puzzle produced by (5) in details (he accepts that some identities are accidental and a posteriori). I only note that his view assumes few things, in particular, the distinction between rigid and non-rigid designators and essentialism as well as admissibility of switching from *de dicto* necessities to *de re* ones. There is not essential difference between (A1) and (A1'), although the latter has the *de re* form (the quantifiers precede the box). However, replacing P by 'necessarily identical with' relevantly uses the *de re* formulation. My proposal consciously ignores all extralogical circumstances except the claim that necessity should be used *de dicto* and as related to the provability. This blocks the passing from $\Box(x = y)$ to $x = y$. Without assuming $x = y$ as independent of $\Box$ or $\vdash$, the conclusion that all identities are necessary does not follow. On the other hand, we can still keep the difference between unconditional (like (A1)) identity validities and conditional ones ((A2), (A3)). Facts of interpretation are of course accidental and a posteriori, for instance, the term 'the capital of UK' could be valued not by London, but other British city, let say, Manchester. However, if an interpretation is fixed, its

consequences are conditionally necessary. Since unconditional identity is a special case of conditional, we obtain a uniform treatment of $=$, independent of the view whether it is a logical constant or not. These conclusions are obvious, if we adopt the objectual understanding of identity.

References

1. Frege, G., *Begriffsschrift, eine der arithmetischen nachgebildete Formelsprache des reinen Denkens*, Neuberger, Halle 1879.
2. Kripke, S., "Identity and necessity", [in:] *Identity and Individuation*, ed. by M. K. Munitz, New York University Press, New York 1971, pp. 135-164; repr. [in:] S. Kripke, *Philosophical Troubles. Collected Papers*, v. 1, Oxford University Press, Oxford, 2011, pp. 1-26.
3. Massey G. J., *Understanding Symbolic Logic*, Harper and Row, New York.
4. Tarski, A., *Introduction to Logic and to the Methodology of Deductive Sciences*, Clarendon Press, Oxford 1941.
5. Tarski A., "What Are Logical Notions?", *History and Philosophy of Logic* 7, pp. 143-154.
6. Wittgenstein, L., *Tractatus Logico-Philosophicus*, tr. by D. F. Pears and B. F. McGuinness, Routledge. London 1961.
7. Woleński, J., "First-Order Logic: (Philosophical) Pro and Contra", [in:] *First-Order Logic, Revisited*, ed. by V. Hendricks, F. Neuhaus, S. A. Pedersen, U. Scheffler and H. Wansing, *λoγoς*, Berlin 2004, 369-399; repr. [in:] [10], pp. 61-80.
8. Woleński, J., „O zdaniach identycznościowych” (On Identity Statements), *Acta Universitatis Wratislaviensis – 399. Logika* 23(2005), pp. 175-195.
9. Woleński, J., "Remarks on Identity across Possible Worlds", *Bulletin of the Section of Logic* 37(3/4)(2008), 273-287; repr. [in:] [10], 243-253.
10. Woleński, J., *Essays on Logic and Its Applications in Philosophy*, Peter Lang, Frankfurt am Main 2011.

Note:

1. This paper uses and extends some considerations in [8] and [9].

There Is No Metaphysical *a Posteriori* Necessity

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Abstract:

In my paper I argue for the claim that even though Kripke is right that the classical two way connection between necessity and apriority does not exist, the *a priori* remains the only way how we know what is metaphysically necessary.

Key words:

a priori, *a posteriori*, identity, Kripke, necessity.

Water is H₂O. Is it simply so? Or *must* it be so? Kripke believes the second: If true at all, it *must* be that water is H₂O. What, however, are his reasons why it *must* be so? Roughly, they are as follows: If two things are identical to each other, then they are necessarily so. The necessity of identity can be proved from the necessity of self-identity and the Leibniz's Law. In his paper Kripke makes use of the well-known Barcan's proof [8, p. 72]:

- (1) $\forall x \forall y (x = y \supset Fx \supset Fy)$
- (2) $\forall x \Box (x = x)$
- (3) $\forall x \forall y (x = y) \supset (\Box (x = x) \supset \Box (x = y))$
- (4) $\forall x \forall y ((x = y) \supset \Box (x = y)).$

However, Kripke's final targets are two claims that enjoy wide acceptance by contemporary philosophers:

There are no contingent identities
and

At least some necessities are known *a posteriori*.

To reach these conclusions Kripke arranged two great conceptual divorces: He separated necessity from apriority and from analyticity [6, p. 4]. On his view, necessity is one thing and knowledge of it is quite another. Necessity does not depend on whether and/or how it is known. Further, for some expressions it is true, that what an expression refers to does not depend on which sense that expression has. Such expressions Kripke call 'rigid designators'. They refer not through their senses but by picking out their referents directly in all possible worlds in which the later exist. In my paper In that what follows I argue for the these that at least one of the Kripke's divorces was unhappy and in order to talk sensibly about metaphysical necessity we need to restore its linkage with apriority, even if it must be looser than it was once thought by Leibniz or Kant.

1. Kripkean Account of *a Posteriori* Necessity

Kripkean view on necessity in general is broadly 'Leibnizian'. Following Leibniz and Carnap, he defines necessity of a statement as its truth in all possible worlds. He accepts that there are at least weak *de re* necessities which are known *a posteriori*. One of the most clear examples of such *a posteriori* known *de re* necessities are statements like

(1) Water is H₂O

or

(2) Phosphorus is Hesperus.

He claims that if the linguistic expressions on both sides of an identity sign are rigid designators then the identity statement asserts not only simple contingent identity, but also *de re* necessity. Since the rigid designators pick out the same referents in all possible in which they have them. So, if the asserted identity is true at all, it is true in all worlds in which the rigid designators on both sides of the identity sign have referents.

In his famous paper 'Identity and Necessity' (1971) Kripke argues for the claim that since every true identity statement is necessary, so every identity discovered in the course of an empirical investigation is necessary too. Thus, according Kripke, there are no true identity statements that are contingent. Therefore, his point is that an identity statement, if true at all, always is necessarily true, no matter how we come into knowledge of it. The necessity is thus free from way of its knowing.

In a very oversimplified manner we can state his argument as follows:

1. All true identities are necessities;
2. Some true identities are discovered *a posteriori*;
3. Therefore, some necessities are discovered *a posteriori*.

In defense of the first premise Kripke says that once we accept that it is necessary that each thing is identical to itself, then we must conclude that if two things are identical to each other, then they are necessarily so. Thus, according to Kripke, in discovering such truths like (1) or (2) we discover not only some contingent facts but metaphysical necessities also.

This assertion goes, however, against our strong intuition that the world could be other way it in fact is. We could imagine that Phosphorus and Hesperus are distinct and water is not H₂O. But if Kripke is right then our intuition goes somewhere wrong. What is the source of the mistake, however? Kripke's answer says, that we are mistaken because we fail to distinguish between metaphysical and epistemological possibility. The first is the way how the world in itself could be or could not be, while the second is the way how we could or could not think of it in the light of our other beliefs. Thus, we are able to imagine things that are not possible in the reality. And the non-identity of Phosphorus and Hesperus or water and H₂O are exactly such things. We due these possibility intuitions alone to deficiency of our beliefs about the subject matter. But once the relevant facts are established we must repair our intuitions on the subject matter.

2. Philosophical problems of *a posteriori* necessity

Michael Dummett writes:

The philosophical problem of necessity is twofold: what is its source, and how do we recognize it? [1, p. 327]

Bob Hale and Aviv Hoffman give the following comment upon Dummett's phase:

His first question plainly presupposes that there is such a thing as necessity; and his second equally plainly presupposes that it is a possible object of knowledge. [6, p. 3]

I agree with them that these presuppositions must be made in order to capture both Dummett's questions. Further, we can suppose that the existence of such a thing as necessity consists in the existence of some modal features which make necessary items distinct from those that are non-modal at all.

Let us consider some examples. First, it may be true *simpliciter*, that

(3) I am sitting.

The statement supposes to express a simple, i.d. non-modal truth. It does say nothing about what is the relation of my present position towards ways it could be. It does say nothing about whether I could be in a non-sitting position instead of being in the sitting one or I could be not and the sitting is the unique position which is available for me. If there is some feature in virtue of which my actual position bears a definite relation towards either contingency or necessity, then it must be true

either that

(4) I am sitting *possibiliter*

or

(5) I am sitting *necessiter*.

I will call a feature which possession explains how a modal statement can acquire its truth value *source of modality*. If the feature explains the truth of a necessary statement, then I will call it *source of necessity*. If the feature explains the truth of a contingent statement I call it *source of contingency*.¹ The question I try to answer later is thus what is the source of necessity in the case of necessary identity statements *de re*. What features, if any, should be added to those that are already there to provide truthmakers for simple statements in order to convert these simple non-modal truths into necessary ones.

If necessity poses a possible object of knowledge how then we reach it? More precisely, how we acquire knowledge of the features which turn a simple non-modal truth into necessary one? I suppose that truth of some simple non-modal statements can be known *a priori* whereas truth of other simple non-modal statements is known *a posteriori*. Examples of later are (1), (2), (3). Examples of the former are

(6) I sit if I am sitting;

(7) Water is water;

(8) Phosphorus is Phosphorus.

All statements known *a priori* seem to be easily convertible into necessary ones even if not in the same way. (6) is necessary but its necessity don't go behind the boundaries of the language. It is only *de dicto*. I am not such that I couldn't stay when I am sitting. The statements (7) and (8) may be necessary not only *de dicto* but *de re* as well. It seems very plausible that water is such that it couldn't be something else than water². And the same is true of Phosphorus. In that what follows I am interested only in *de re* necessity which involves identity assertions. That is, my question is what is the source of necessity of true necessary *de re* identities.

Consider (7). What is it that permits us to convert it into a necessary statement? The simplest answers might seem to be its *analycity*. (7) is necessary because it is analytical. The problem with that answer is that it is hard to see how analycity, may it be what it will³, is able to explain more than only a *de dicto* necessity. Because analycity of (7) amounts to synonymy of the senses of two alike expressions on both sides of the copula: whatever water is it is that what it is. Analycity is a linguistic or semantic feature of statements, not metaphysical one. It characterizes relationships of senses of linguistic expressions. [7, p. 17, 40 ff] And thus analycity is not able to account for *de re* necessity which is a metaphysical feature of things, rather than linguistic one. Therefore, we may conclude, that the analycital *a priori* as such cannot be source of metaphysical necessity. At this point, my conclusion stays in no contradiction to Kripke's one. He also holds that neither apriority nor analycity are reliable routes to metaphysical *de re* necessity.

As next we can state the hypothesis that sources of metaphysical *de re* necessity are essences of things. In the contemporary philosophy this idea is widespread enough. Kit Fine expresses the thought as follows:

... any essentialist attribution will give rise to a necessary truth; if certain objects are essentially related then it is necessarily true that the objects are so related (or necessarily true given that the objects exist). However, the resulting necessary truth is not necessary simpliciter. For it is true in virtue of the identity of the objects in question; the necessity has its source in those objects which are the subject of the underlying essentialist claim.[4, p. 7]

Consider again (1). According to the hypothesis stated above, if (1) does capture the essence of water we could see it as the source of metaphysical *de re* necessity for:

(9) Water is metaphysically necessarily such that it is H₂O.

Granted that we know (1) *a posteriori*, how then we are certain that (1) is about water's essence and not about something else?

For consider also the following statement about water:

(10) Water is a liquid.

Does it express an essential truth about that stuff?

We might observe how under different conditions an aggregate of H_2O molecules which is big enough equally takes form either of a solid, or of a liquid, or of a fluid. At the same time it seems impossible for a liquid neither to be a solid nor a gas, if *being a liquid* is an essential feature of the liquid in question. Naturally, we could say that *being a liquid* is not a part of water's essence, but is only one of its modes. On the other hand, *being a liquid* seems to be a part of water's definition and thus constitute a part of that what is it to be water, at least if we are guided by Kit Fine's idea that essence of a thing and its definition are at bottom the same. [4, p. 11]

Once confronted with a case as mentioned above we might ask the following question: what in our observation of water's behavior allows or interdicts us to include or exclude the property of *being a liquid* from water's essence? Granted *being H_2O* is a part of water's essence and granted this is a fact which was established *a posteriori*, we might doubt that water has the property of *being a liquid* and that of *being H_2O* both essentially. Thus, we have to decide which of the two is essential one and which not. I don't see how the decision could be made *a posteriori* alone. Moreover, even if we could say that water's *being H_2O* is grounded on a better observation which reveals the essence of that stuff somewhat deeper than water's *being a liquid* which is nothing more than a superficial sight upon that what water really is, we will have then to answer the following question: How the expression 'Water is H_2O ' does exactly mean?

Even at first look we have here many options. First, it might mean that a water molecule is identical to H_2O . Second, that a water molecule is composed of two hydrogen atoms and one oxygen atom. In either case we read the expression as if it were about a chemical thing – a certain molecule, but on the first reading, it asserts something about its identity, whereas, on the second, it asserts something about its composition. We may, however, conceive the same expression as if it were not about water molecule at all. For example, it could mean the stuff in my glass which I use to drink each morning. In that case it would, perhaps, asserts of it that it is composed mostly of water molecules. Or something like this. Either way, in every case the underlying empirical observation would be the same. And we should consider the question like whether many things do compose one unique thing when they come in the vicinity of each other? Or, is composition essential to that what it is composition of? Is actual identity of a thing essential to that thing? Are composition and identity the same relation? None of the questions seems to be answerable *a posteriori*. If there is any way to answer them, it is plainly *a priori*.

3. Necessity and the A Priori

In my view there are some things we should ask about Kripke's putative examples of *a posteriori* metaphysical necessity. Many writers pointed to the circumstance that Kripkean proof of metaphysical necessity of empirically discovered identities is not possible without such principle like necessity of identity or Leibniz' Law, which are clearly *a priori*. [9, p. 742, 4, p. 11, 5, p. 147–164] Thus, this proof gains its strength from purely *a priori* statements. On the other hand, almost nobody of the contemporary philosophers doubt that Kripkean examples are examples of genuine metaphysical necessity. If so, one could defend the following thesis:

Some metaphysical necessities hold partly in virtue of their *a posteriori* contents.

Even if we could not reach any metaphysically truth directly through empirical investigation, we could gain some important parts of it this way. The claim is not so innocent as it might appear to the first sight.

Consider mind-body problem. Suppose, we have empirical evidence that this mental state S is identical to that physical state S' . Does it amount to a metaphysical necessity that $S = S'$? If yes, then having enough analogous evidences we could by induction infer that all mental states of the type to which S belongs are identical to the type of physical states to which S' belongs. And that all this is a metaphysical necessity. Following this way further we may step by step reach the conclusion that all, or at least all known to us, mental states are identical to physical states and this conclusion will hold with the strength of metaphysical necessity. Thus, it would be allowed to solve the mind-body problem empirically. Many philosophers are very close to believing in such an

option. Hence, the existence of metaphysical necessities *a posteriori* is welcome to them. Unfortunately, this option is unavailable to us, if we look at the problem of source of metaphysical necessity more carefully. I see at least two considerable embarrassments which impede us to reach metaphysically necessary truths by means of an empirical investigation. First problem is why should we think that a necessary truth which has an *a posteriori* content as its part does express metaphysical necessity rather than natural one. Second problem is, why should we think that the necessity of a necessary truth which has an *a posteriori* content holds in virtue of it's *a posteriori* content rather than in virtue of something else?

Consider the first problem. Suppose that there is a natural order in our world. We might think it as a set of laws which govern over all and only concrete particulars existing in that world. Call things which are governed by such laws natural things. It is plausible that water is one of the natural things. All what is naturally possible in our world is determined by the laws of that world. It does not mean that there is only one way in which our world could be. Because there could be many non-identical sets of natural things that exist in it as well as the laws themselves could be indeterministic. Thus, there is a sensible non-trivial notion of natural necessity for our world. We could say that something is naturally necessary in our world iff it is *in virtue of* laws of our world that it is *not possible* that it is not the case in our world. Natural necessity could be thought either as necessity *sui generis* à la Kit Fine or as appropriate restriction or relativisation of metaphysical necessity. [3] In each case the natural necessity could be different from the metaphysical necessity at least intensionally. Only then the naturalistic claim that natural necessity coincides with metaphysical necessity would be a non-trivial one. Suppose further that the natural things in our world have natural essences. We could say that something is a natural essence of a thing iff its possession by that thing is due entirely to natural laws. In somewhat other way we could determine the natural essence of a thing as a set of properties which that thing has in virtue of natural laws. This whole set of such properties constitutes natural identity of a thing. Natural essences could be different from metaphysical ones. For example, water as natural thing could have the property of *being identical to H₂O* as a part of its natural essence. Water *as such* could have a property of *being a substance* as a part of its metaphysical essence. Then a philosophically interesting question about the relationship between metaphysical essence of a thing and its natural essence arises whether they coincide or not.

Take the water example again. We grant that it is necessary, that water is H₂O. Which kind of necessity, however, does the phrase express? According to one reading, it could mean something like

(11) It is necessary, that under certain conditions two atoms of hydrogen and one atom of oxygen compose one water molecule.

According to another reading, it could mean something like

(12) It is necessary, that water is such that it is composed of two atoms of hydrogen and one atom of oxygen.

I think, it is clear that the expression 'it is necessary that' in (11) means natural necessity, whereas in (12) it means metaphysical necessity. In my view, the first reading is preferable in that case of water. But even if (12) is the right one, it seems to hold in virtue of some *a priori* principles like

Composition of a thing is essential to it
and

Essence entails metaphysical necessity.

I believe, from all that we must come to the conclusion that even if natural and metaphysical necessities coincide, the argument for that coincidence must be *a priori*.

Let us turn to the second problem I indicated above. Does the necessity of a necessary statement that has an *a posteriori* content as part hold in virtue of that *a posteriori* content?

Look at the water case again. Let us now understand 'is' as 'identity' rather than as 'composition'. It seems that the best candidates for being source of metaphysical necessity are essences of things which are the parts of the non-modal proposition that has to be converted into the corresponding necessary one. In our water case there three essences which could be the source of

necessity: essence of water, essence of identity relation and essence of H₂O. The further question is what feature of a thing could count as its essence? Following Kit Fine, we could suppose that essences of things are their identities or, may be, haecceities. It seems, however, that there is no empirical way to decide whether a feature of a thing belong to the essence of that thing. Moreover, in the water case it is the essence of the identity relation seems to be the all best candidate for being source of necessity, because it has the property of holding necessarily, if it holds at all. Perhaps, necessity of identity is not indisputable. But anyway, it is an *a priori* affair to decide whether the identity relation is necessary or not. The question could not be answered in an empirical way. The same goes for haecceity of water or H₂O. We could not say on the basis of any empirical investigation of these things whether a certain feature of them is their haecceity. Consequently, the essence of whatever part of the proposition 'Water is H₂O' may be the source of its necessity, it is always an *a priori* inquiry which helps us find an appropriate answer to the question.

4. Conclusion

Which moral then have we to draw?

I think that if there is such a thing as metaphysical *de re* necessity then it exists not in virtue of apriority or analyticity. Because both are not appropriate features of this modality as such. Necessity is rather a relation of a thing's essence to ways of its being. Here Kripke was right. Where he was mistaken, however, is the way we know necessity. The *A priori* is the only way to know what is metaphysically necessary. No metaphysical necessity could be known through empirical investigation. The Kripkean divorce of necessity and apriority turned out to be all too hasty. A repeated union is needed, even if that time it should be more loose than in the times of Leibniz and Kant. A two way connection between necessity and apriority indeed does not exist. Because it is not true that a proposition is necessary if it is *a priori*. The real connection is much subtler. Nonetheless, it is very tough and substantial. Apriority remains a necessary, even if not sufficient, condition not for the necessity itself, but for the knowledge of it.

References

1. Dummett M. "Wittgenstein's philosophy of mathematics", *Philosophical Review* 68 (1959): 324-348.
2. Fine, K. "Essence and Modality", [in:] *Philosophical Perspectives* 8 (ed. J. Tomberlin), 1994: 1-16.
3. Fine K. "Variety of necessity" [in:] *Modality and Tense: Philosophical Papers*. Oxford University Press, 2005: 235-260.
4. Fine K. "What is metaphysics?", [in:] *Contemporary Aristotelian Metaphysics* (ed. T. E. Tahko), Cambridge: Cambridge University Press, 2012: 8-25.
5. Garcia-Encinas M. J. (2012) "On Categories and A Posteriori Necessity: A Phenomenological Echo", *Metaphilosophy*, 43: 147-164.
6. Hale B. & Hoffman A. (eds.) *Modality: Metaphysics, Logic and Epistemology*. – Oxford University Press, 2010.
7. Katz J. *Sense, Reference, and Philosophy*, Oxford University Press, 2004.
8. Kripke S. Identity and Necessity, [in:] Kim, J. & Sosa, E. (eds.), *Metaphysics: An Anthology*. Oxford et.al.: Blackwell Publishing, 1999: 72-89.
9. Peacocke Ch. "The A Priori", [in:] Jackson F, Smith M. (eds.) *The Oxford Handbook of Contemporary Philosophy*. Oxford University Press, 2005: 739-766.

Notes

1. In this paper I omit impossible statements out of consideration.
2. Think, however, of the biblical story about the Marriage at Cana (John 2, 1- 11), which while seeming perfectly intelligible, presupposes that at least God is able to turn water into wine.
3. Today there are two main positive, i.d. non-skeptical, accounts of analycity: Frege-Carnapean, or Logical analycity, and Locke-Kant-Katzean, or Mereological analycity. According to Frege-Carnapean account, a statement is analytical iff its truth-value determined by definitions of the terms involved in the statement and logical laws. According to Locke-Kant-Katzean account, a statement is analytical iff its truth-value is determined only by meanings of terms involved in that statement. See more in Katz J. *Sense, Reference, and Philosophy*, Oxford University Press, 2004, p. 40 ff.

Abstract Incompleteness Theorems and Their Influence in Methodology

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Abstract:

Kolmogoroff's complexity, Chaitin and Gödel's incompleteness theorems are considered commonly relatively to a fixed coding of objects and to a standard notion of algorithms. In essence all they are independent from almost all properties of concrete theories, algorithms and codings. So stable and general results are to have deep methodological, philosophical and even theological consequences. Here we consider their abstract form and speculations which can be derived from them and partially from modern computer science, IT practice and physics. Main ones are the following:

A new proof of Kant's Third antinomy and of Parkinson's Law of committee, relations to cooperative creative activity, multi-language programming, benevolence to other's views, dilemma of deism-atheism, and finally methodological approach to theology.

Keywords:

Kolmogoroff complexity, Chaitin theorem, Gödel theorem, Kant antinomy, Parkinson law, God, Science, truth, destination.

1. Abstract Incompleteness Theorems

1.1. Abstract Structures of Computability

Gödel theorem of incompleteness and Chaitin theorem of incognizable are affecting many philosophical speculations and many branches of Western philosophy. Due to their importance it is reasonable to clarify whether they deal with one kind of precisely defined structures or with a wide class of such (and maybe not fully defined) structures. So we start with a popular outline of proof of their very common and abstract nature making them in essence basic and inevitable restrictions of precise thinking. A precise formulations and proofs are given in [1, 2]

A *system of computability* works over any finite or infinite set of atoms. It transforms lists constructed basing on these atoms. List is so general and flexible information structure that non-natural encodings of other objects (like to Gödel numbers) are not necessary.

For example string of letters 'abca' can be represented by a list (a b c a string). A logical formula can be represented by a list like to (all x (exists y ((2 * x) = y))). Natural numbers are expressed as lists of the form (Nil ... Nil) where Nil is an empty list: Nil=()=0.

In a system of computability some atoms are declared as *functional* ones. If a list ends by a functional atom it can be computed. This is like to a practice of the functional programming languages Lisp, Haskell and so on. The functional atom defines an action; other members of the list are the arguments of this action. For example ((a b c) (b a) concat) is computed into (a b c b a) if concat is a function of list concatenation. There can be only a finite list of functional atoms.

We assume only basic elementary transformations of lists. Because functional atoms are atoms also we assume also elementary operations over functions which not lead us out of the space of functions generated by expressions using elementary functions. Because those functionals can compute to functions or functionals some of resulting lists are called *functional lists*. If the last member of a list is the functional list and number of other members is equal to the number of arguments of the function then this list is called *convertible* one and can be computed. More precisely list is *convertible*, if there is a subexpression which is not functional and of the form $(E1 \dots En F)$, where number of arguments of F is n .

For example if $comp$ is a functional of composition for unary functions and f and g are function lists then $(a (f g comp))$ is the convertible list which computes to a result of sequential application f and g to a .

Attention! We write “a result” because we do not assume that each function is deterministic. Our basic functions are deterministic but we allow indeterministic functions and functionals also.

We allow any finite number of additional elementary and high-order functional atoms. So a system of computability can be very weak or very strong. It is not necessary Turing complete or Turing computable. The only condition is that each function has a well defined computational semantics (not necessary algorithmic). Thus we defined a kernel language for different kinds of algorithmic and non-algorithmic computations (e.g. hyperarithmetic or computations on an algebraic structure). Lists in a system of computability are also called *expressions*.

We demand the following proposition holds.

Proposition 1 (limited λ -abstraction). Let us enrich our language by variables $x_1, \dots, x_n$. Then for any list $E[x_1, \dots, x_n]$ containing variables $x_1, \dots, x_n$ can be constructed a functional FT such that $(E1 \dots En FT) = E[E1, \dots, En]$.

Here $=$ is understood as “sets of possible values of left and right parts are equal”.

If a system of computability contains a functional atom *turing* it is called interpretative one, if it contains *eval* it is called Turing one. It is called strong Turing one if it also contains a functional atom *search*. Computational semantic of these two functionals is defined as follows.

$(E1 E2 E3 \text{ turing})$ computes $E2$, which is to be a functional, and then performs $E3$ steps of its application to $E1$ ($E3$ is to have a number value) and gives a list $(E4 E5)$, where $E4=0$, if computation had been finished on or before step $E3$, and 1 else. $(E1 0 E3 \text{ turing}) = (1 E1)$.

$(E1 E2 \text{ eval})$ computes its arguments, the second argument is to be a functional, and then apply this function to the value of the first argument. $(F F1 \text{ search})$ finds such tuple of values of arguments for F , for which F is equal to 0, and applies $F1$ to a found value. Numbers of arguments of functions are equal.

Proposition 2. There holds a fixed point theorem in each Turing system: for each functional F there is such E that for all $E1$ $(E1 E F) = (E1 E \text{ eval})$.

Proposition 3 (Turing completeness). Turing systems allow expressing any partial recursive function.

eval is definable through *turing* and *search*. It is called *a universal function*, *turing* is an interpreter, *search* is a search operator, it can be whether indeterministic or deterministic. No other dependencies hold for these three operators. Primitive recursive functions have an interpreter without search and universal function. Recursive schemata on real numbers and their lists with a signature $\{0.0, 1.0, =, >, +, *\}$ have universal function and interpreter but no search. Hyperarithmetical functions on real numbers have no search and no interpreter, only a universal function. Adding search we get no interpreter. Adding search to initial elementary functions gives no interpreter and no universal function.

1.2. Generalization Of Algorithmic Complexity

Definition 1. *Complexity* of an object relatively to a system of computability is a minimal length of an expression which evaluates to our object. If a system is Turing one, complexity is called Kolmogoroff one. Complexity of an object x in a system Σ is denoted $(x K_\Sigma)$.

If system is defined by a context Σ is omitted. Let lh be a function computing length of an expression (small details like whether spaces are counted and so on are irrelevant here; it is sufficient that length is strictly increasing and almost additive via concatenation).

Definition 2. Let there are two computational systems Σ_1 and Σ_2 . Coding $CODE[a]$ of the language of one system inside language of other is *regular* if $(CODE[a] \text{ } lh) \leq k \cdot (a \text{ } lh) + C_1$. C_1 is a constant, k is a *coding factor*. Σ_1 is interpreted in Σ_2 , if there is a regular coding and a function Int and there exists n such that $(CODE[E] \text{ } Int \text{ } turing_2) = (0 \text{ } CODE[a])$ iff $E=a$.

Σ_1 is translated into Σ_2 , if there is a regular coding and a function $Trans$ such that $(CODE[E] \text{ } Trans \text{ } eval_2) = CODE[a]$ iff $E=a$.

Theorem 1. (Kolmogoroff theorem) If Σ_1 is interpreted or translated into Σ_2 and k is a coding factor, then $k \cdot (a \text{ } K_1) \leq (CODE[a] \text{ } K_2) + C$.

This theorem generalizes up to wide class of systems and codings (including Turing-incomplete and non-algorithmic) a theorem of Kolmogoroff on invariance of complexity up to additive constant.

1.3. A Generalized Chaitin Theorem

Let there is a theory **Th**, having definable predicates: unary “To be a natural number” ($x \in \mathbf{N}$) and two binary: $=$ and $<$. Let **Th** has constants $0, 1 \in \mathbf{N}$ and functions of natural numbers (maybe also definable) $+$, $*$, $\uparrow$. The last function is the power function. Elementary arithmetical formulas are relations of two expressions in this vocabulary. Then we say that this theory contains natural numbers.

Let there is a full Turing system Σ with functionals which can to test whether this list is a proof of a given formula in some regular coding, to extract a proved theorem from a proof code and to substitute an object of Σ (not necessarily a number) for a free variable of a formula and to compare two formulas textually.

Definition 3. A theory is Chaitin-correct w.r.t. Σ if the following notions are expressible a notion $(E \text{ } E1 \text{ } eval) = a$, a function $(a \text{ } lh)$ which computes the length of an expression; all true formulas of the form $(a \text{ } lh) = n$ are provable; all closed true elementary arithmetical formulas are provable; and no closed false formula of the form $(E \text{ } E1 \text{ } eval) \neq a$ is provable.

Each Chaitin-correct theory is consistent. A simplest such theory Ar_0 can be given by the following axioms:

$(x+0=x)$; $(x+(y+1)=(x+y)+1)$; $(x*0=0)$; $(x*(y+1)=(x*y)+x)$; $(x\uparrow 0=1)$; $(x\uparrow(y+1)=(x\uparrow y)*x)$.

To prove Chaitin-correctness of this minimal theory we are to use a relatively sophisticated encoding like to Smullian's or Gödel's. But this becomes not necessary in a bit less minimalistic constructions.

Theorem 2. (Generalized Chaitin theorem) There is a number C (Chaitin's constant) in any Chaitin-correct theory such that $(a \text{ } K) > C$ is not provable for any a .

Proof.

A formula expressing $(E \text{ } E1 \text{ } eval) = a$ is denoted $R(p, x, a)$. Then a statement $(a \text{ } K) > C$ can be formulated as follows: $\forall x \forall p ((x \text{ } p) \text{ } lh) < C+1 \supset \neg R(p, x, a)$. If $(a \text{ } K) < C+1$ holds, then this formula is not provable inside **Th**, because else a false statement

$$(((x0 \text{ } p0) \text{ } lh) < C+1 \ \& \ \neg R(p0, x0, a))$$

would be provable and thus a false formula $\neg R(p0, x0, a)$ for some $((x0 \text{ } p0) \text{ } lh) < C+1$ also. Let show this and by the way construct a Chaitin's constant.

Let a functional K find for each C a proof of a formula $(a \text{ } K) > C$ by brute force and if such proof is found gives a . Let the length of a code for this functional be k . Let the quantity of different atoms in our system be m . Then there is such C_0 , that $m \uparrow C_0 > k * C_0$. This C_0 can be taken as a Chaitin's constant. Let $(a \text{ } K) > C_0$ were provable for some a . Then K would find such a_0 . But really $(a_0 \text{ } K) \geq C_0$ and thus $((x0 \text{ } p0) \text{ } lh) < C_0+1 \ \& \ \neg R(p0, x0, a_0)$ is not provable for some $p0, x0$.

But $\forall x \forall p ((x \text{ } p) \text{ } lh) < (C_0+1) \supset \neg R(p, x, a_0)$ implies $((x0 \text{ } p0) \text{ } lh) < (C_0+1) \supset \neg R(p0, x0, a_0)$. $((x0 \text{ } p0) \text{ } lh) < (C_0+1)$ is provable by correctness, therefore is provable $\neg R(p0, x0, a_0)$. Contradiction.

Q.E.D.

This form of Chaitin's theorem does not demand computability of a system complexity is defined w.r.t. It uses search function essentially. It can be applied also for systems with infinite basic data type but with finite base of explicitly given atoms. Then complexity of some objects can be infinite (e.g. π in a system for algebraic operations on real numbers).

1.4. A Generalized Gödel Incompleteness Theorem

Now we consider and generalize the Gödel incompleteness theorem in the form of Rosser [3]. Here are some auxiliary definitions.

Definition 4. Restricted quantifiers are formulas of the form

$$\forall x((x \text{ lh}) < n \supset A(x)), \exists x((x \text{ lh}) < n \ \& \ A(x)).$$

A formula $P(x)$ is *limitedly correct* in the theory **Th** if from provability of $\exists x((x \text{ lh}) < n \ \& \ P(x)) \vee B$ follows provability of $P(a)$ for some $(a \text{ lh}) < n$ or provability of B itself.

Definition 5. A theory is *Gödel-correct*, if a predicate $<$ is expressible for natural numbers; all closed true formulas of the form $(a \text{ lh}) < n$ are provable; there is some coding for formulas; there is a formula expressing “ p is a proof of $A(a)$ ” $\text{Proof}(p, \text{CODE}[A], a)$; there is a functional to compute code of negation of a formula by its code Neg ; if $A(a)$ is provable, then $\text{Proof}(p, \text{CODE}[A], a)$ is provable for some p ; a weak Gödel rule

$$\frac{\text{Proof}(p, \text{CODE}[A], a)}{A(a)}$$

is admissible and $\text{Proof}(p, \text{CODE}[A], a)$ is limitedly correct for all A, a .

Theorem 3. (Abstract Gödel theorem) If a theory is Gödel-correct it is incomplete.

Proof.

Consider a formula

$$\forall x((\text{Proof}(x, z, z) \supset \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, (z \text{ Neg}), z)))) \ \& \ \exists x((\text{Proof}(x, (z \text{ Neg}), z) \ \& \ \neg \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, z, z))))$$

Substitute in it its code R . Then if the formula

$$\forall x((\text{Proof}(x, R, R) \supset \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, (R \text{ Neg}), R)))) \ \& \ \exists x((\text{Proof}(x, (R \text{ Neg}), R) \ \& \ \neg \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, R, R)))) \quad (\text{Rosser})$$

is provable, we take a_0 with provable $\text{Proof}(a_0, R, R)$. Due to limitedly correctness of Proof and by the first conjunctive subformula there is such $(a_1 \text{ lh}) < (a_0 \text{ lh})$, that $\text{Proof}(a_1, (R \text{ Neg}), R)$ is provable. Then by a weak Gödel rule a negation of (Rosser) is provable and our theory is inconsistent and proves everything. So it is not Gödel-correct.

If a negation of (Rosser)

$$\exists x((\text{Proof}(x, R, R) \ \& \ \neg \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, (R \text{ Neg}), R)))) \vee \forall x((\text{Proof}(x, (R \text{ Neg}), R) \supset \exists y((y \text{ lh}) < (x \text{ lh}) \ \& \ \text{Proof}(y, R, R))))$$

is provable then there is such b_0 for which $\text{Proof}(b_0, (R \text{ Neg}), R)$ is provable.

From first disjunctive part follows $\exists x((x \text{ lh}) < (b_0 + 1) \ \& \ \text{Proof}(x, R, R))$. Applying limitedly correctness we get provability whether (Rosser), which is contradictory, or the second disjunctive part. Then we get a contradiction analogously to the first part of proof. Q. E. D.

So to exclude almost all positive assumptions in incompleteness theorem it suffices only to improve a construction of an undecidable statement.

2. Philosophical And Methodological Consequences

2.1. Algorithmic Randomness and Kant's Third Antinomy

The Kant's Third Antinomy (of Freedom) can be substantiated precisely if complexity of a human is lower than complexity of the Universe. We have stated that any formalism has limits such that upper them it cannot state a complexity of an object and thus cannot correctly comprehend and

understand it. Thus any argumentation with complexity upper than Chaitin's limit for a person is treated as completely chaotic and illogical. But this is not the worst case. If such a person tries to comprehend the arguments by cutting out all which cannot be placed in his/her head he/she gets an illusion of understanding together with completely wrong image of the percept.

Chaitin [5] noted out that now existence of unknowledgeable is well substantiated and even proved. Each position based on supposition that human mind is omnipotent in principle is not even an opinion now. Our generalization of Chaitin theorem shows how weak premises are sufficient for Chaitin's limit is existent. We do not need here a claim that human is a finite system which had been used in earlier demonstrations. This together with an observed harmony of the world substantiated deism in very high degree [6]. In the same time this shows that it is impossible to prove or to refute existence of God.

For finer methodological consequences it is reasonable to accept finiteness of a human (as for example in [9]). The Universe is incognizable as a whole because complexity of the Universe is much higher than one of a human and of the humanity (even in supposition that joining humans join only knowledge but not their ignorance). But incognizable can sometimes be partially appreciated. It is known that objects with big Kolmogoroff complexity are comprehended as random.

Kolmogoroff studied algorithmic randomness for infinite sequences (complexity of initial segment of a sequence will be same as its length up to additive constant). We are to define randomness of a finite object from the point of view of Chaitin's limit and his considerations in [6,7]. This is randomness relative to a concrete object or subject processing information.

An object is random for a processor if its complexity is larger than processor's Chaitin's limit.

Now we'll prove a proposition equivalent to Kant Third Antinomy [10] and even in more strong form, expressing it in the language of current science.

Human cannot state whether our Universe is deterministic or there is a necessary randomness in it.

Let the Universe be deterministic. Then a complexity of the algorithm initialized during world's creation is higher than Chaitin's limit of humanity. Thus humanity cannot comprehend a Word's idea as a whole and complete entity. Deterministic world is understood as random one.

We must state a warning here. We are not creationists. World creation could be a natural process for example as garbage of a super-civilization during re-creation or transformation of its own World (S. Lem: *From the Einsteinian to the Testan Universe*. In [11]).

Let our World be indeterministic. If we were proved this we were proved that complexity of our World is higher than Chaitin's limit of our civilization. This is a contradiction.

Thus problem whether our Universe is deterministic is a pseudo-problem from the point of view of pure exact knowledge. We are free to choose a theory which in the moment is a best fit for «practice» and is a better representation of objects in view.

Therefore it is unacceptable to advertise results of our science as «scientific truths». They are to be re-verified by an alternative theory. This is a strong opposition for postmodernistic «tyranny of truth». We cannot lay our responsibility on arms of Science or God.

2.2. Parkinson's Law

Parkinson's law of committee (decision of committee is more moronic than decision proposed of its stupidest member) can be proved precisely. Let there is a committee which is to work out a decision understandable for all its members for each could meaningfully vote *yea* or *not*. In this case Chaitin's limits of committee members are to be reduced to minimal one because else some of members cannot understand a proposal. So a weak Parkinson's principle is substantiated:

Weak Parkinson's law:

Decision of a committee is no more adequate that one which could make the least competent of its members himself

But the reality is cruder. Each committee member has different competencies in different domains. So we need to introduce a matrix of limits. If two limits of persons are C_i and C_j , complexities of translations from one system of notions into another are K_{ij} and K_{ji} , then maximal complexity of a decision of each of them understandable by both is $C_{ij} = \min C_i - K_{ji}, C_j - K_{ij}$: a limit of i -th person for understanding of j -th. Thus even not taking into account non-uniformity of knowledge inside a Chaitin limit we get the following upper bound: $\min_{i,j} C_{ij}$.

We substantiated the following

Strong Parkinson's law:

Decision of a committee is more moronic than a decision which could make the most moronic of its members himself

In Venice and Rome important decisions were delegated to a truthful person which had been made fully responsible for its realization and consequences...

2.3. Chaitin Limit and Paradox of Inventor (Orevkov Theorem)

There is at least one more quality of mind orthogonal to brute force which can lead to relatively large Chaitin's limit. This is ability to master complex notions.

Orevkov theorem (1968):

An indirect proof in logic can be in the tower of exponents times shorter than any direct one.

Orevkov's theorem is a precise partial case of a general paradox of inventor formulated by Gy. Polya:

To prove a simple statement we are often to use complex intermediate notions. To prove a weaker and "simpler" statement can be much harder than to prove more strong and complex one. Gy. Polya pointed out and partially explained this paradox w.r.t. inductive proof. Orevkov substantiated that it is a fundamental property of thinking.

Interrelation of Chaitin and Orevkov theorems yields that high level person can make things which cannot be understood by plain thinkers but to implement his/her insights plain thinking is often necessary. Using high order notions we can jump far away behind Chaitin's limit of crawling persons. This substantiates a genial insight of D. Hilbert that ideal notions are necessary to obtain non-trivial practical (real) results.

American scientist M. Furman wrote (private communication discussing my preliminary notes on Chaitin's limit):

"Non-equivalence (not considering purely theoretical notion of Kolmogorov complexity, but from the point of view of real application) is defined by resources: size of memory and execution time.

Theoretically we have two binary properties: is memory finite or is time finite. But seeing one step deeper we understand that there is a uniform restriction for some class of examples".

These arguments do not disturb our basic considerations and only show that real situation is even more fine and interesting. It is known that primary resource of human defines his/her logic (linear logic is logic of money, intuitionistic one is logic of knowledge, nilpotent one is logic of time and so on). Of course it can restrict Chaitin's horizon even more substantially than Kolmogoroff complexity.

M. Furman also proposed an example showing interconnections of Chaitin's limit with inventor's paradox. If a person mastered a high-level method he can say something like to Furman's objection: "It is very easy to construct a translator having the precise definition of a language" But method of formal semantics itself cannot be treated as a simple one. And it is known how hard is to write out a formal definition of a semantic.

Evgeny Kochurov pointed out (private communication) that usually those who cannot comprehend complex notions but have a big operative memory can build long and relatively complex first-order compositions. Those who excellently appreciate methods can find excellent critical points but poorly analyses a crawling process how to go from one critical point to next one. So those two are complementary and can excellently assist one another if each person is used according to his/her strong sides. So we transferred to a problem how to avoid Parkinson's law.

2.4. Consequences for Organization of Creative Work: How to Avoid Parkinson's Law?

There is an interesting example which seems to be a strong counterexample to Parkinson's Law. Each bee, termite or ant acts like to finite automaton with a fixed program and low memory. Nevertheless a general behavior of nest becomes very complex and adaptive. Moreover ants for example demonstrate more complex forms of integration and system behavior. Remember ant empires joining in the single net thousands of nests which have intensive exchange of information, people and genetic material (trade points and exchange of nymphs).

We apply here an analogy from logic. Von Neumann's theory of self-reproducing automata shows how to compose an upcoming system from uniform units with extremely simple behavior. Thus a good organization of morons which cannot understand even loops can generate recursions and high level constructions. How is it possible? It is because cooperation itself is performed by strict simple automata rules.

This analogy is used in neuron nets in such domains as pattern recognition in cases when there are no precise algorithms. Well trained neuron net mistakes sometimes but rarely. And nobody knows why.

Ideology of crowdsourcing tries to transfer this experience into human society. But as for neuron nets here we get no creativity¹. How to introduce it?

As usually direct and obvious decision --- to make automata stochastic or indeterministic --- fails here. Such approach to creation process is fantastically ineffective.

So we come to a tough consequence for human collectivities. Committee consisting from equal and free creative persons is impotent. Potent can be at least two-level structure. Interactions are strictly formalized on first level and for connections between first and second level. In contrary interactions on second level are bounded by clear and ruthless rituals but never formalized. They are diminished to a reasonable minimum. Upper level is responsible for creative decisions and lower for their realization. It is often possible to implement an idea inside a rigid structure but never is possible to get a new idea here.

We have here another “counterexample”: freesofters. This seems to be a conglomerate of free creative individuals which interact very informally. But this is not the case. They curse and laud one another very informally but their interactions in coding, bug processing, and documentation and so on follow strict rules. So I cannot say that they are “free persons” in vulgar sense of this word. They are free individuals having real goals and values and voluntarily sacrificing some “freedoms” for those high valuables. They can be an embryo of a structure which can save humanity and some real achievements of current ill civilization after its inevitable death.

And now dive in cold water. A community of freesofters can be so effective because almost all they are involved into really non-creative problems of coding according to existing algorithms and architects, debugging and developing earlier projects. But this community has also an ecological niche for really creative persons.

Warning. A society based on freesofters-like libertarian principles will ruthlessly apply “measures of humanitarian defense” (see e.g. a social fantasy of A. A. Rosoff “Confederation Meganesia” [11]) and suppress minorities which wish to claim their rights in manner restricting other people's rights and common values. It may be necessary to survive against mindless hordes of “free vultures”.

Furthermore collective intellect of best algebraists allowed solving a problem of classification of finite groups [12]. But interaction of professional pure mathematicians is so deeply ritualized² that this example is a verifying example for us.

These examples allow us to make principle of committee more precise. Committee must elaborate a decision. Such decision will inevitably be a compromise e.g. a mixture of unpleasant and useless. Creative persons try to find a solution. They do not try to cut it according to lower level of their understanding. In contrary, people develop a nice idea of other's even they do not appreciate it as a whole and often find new aspects of it. So a good organized creative storming can lead to valuable results. High level people know how useful is a discussion of equal in spirit and mind persons (but not those nominated by an institution).

Collective creative work is development and transformation of new ideas without “full comprehension”. How to increase effectiveness of this storming?

1. Sacrifice sacred cows.
2. Make hidden conceptual contradictions visible.
3. Don't pronounce “universal and indisputable truths” (Благоглупости in Russian; I don't know an analogy in English).

All these three points contradict to politcorrectness and other liberal taboos.

2.5. Three heads of stupidity

In preceding discussion we have used three terms characterizing main sources of “high-minded stupidity” which we can often observe. A person who has a well developed reasoning power makes completely inadequate and often horrible and deadly propositions and claims when applying his/her mind to living situation [16].

First of all we must point out that any living notion and any living system are informalizable. This effect usually is ignored and hidden by “sweet and politcorrect” term “hardly formalized”. But really the living notion simply breaks borders of any its formalization.

For example many people tried to give a precise definition of love. If this definition becomes precise then love becomes reduced to courtesy. This notion allowed such phenomena as court of Marguerite de Navarre resulting in statement: “There cannot be love of husband and wife.” William Shakespeare brilliantly refuted this in his “Romeo and Juliet”.

N. Belyakin (Novosibirsk mathematician and logician) approx. in 1976 pointed out that any formalization of a complex, important and living humanitarian notion immediately tends to be refuted by artists and writers. And they always succeeded in this “destructive action”. Moreover a like situation arose in Mathematics. Any formalization of (say) arithmetic helps us to construct an example of statement which is not covered by this formalism (see Gödel theorem earlier in this text).

So any formalization turns notion into precise but dead terms (corpses of notions). It is adequate only when its numerous explicit and implicit suppositions are valid. Thus *it is adequate only in some state of world, for some goals and when some values are accepted as main ones*. It is to be replaced by other formalization or elsewhere lead us into a mortal deadlock. Moreover in cool and clever society this change is to be done when the current formalism is as yet effective but leads to negative cumulative effects. For example such two formalisms as the system of scientific grants and world of virtual money and consumerist economics thinking society is to replace today (if not yesterday).

Different formalisms are mutually inconsistent. So we are to make choice and not seek a quasi-compromise. Theoretically all possible formalizations of a system of notions form a system of theories in which each theory except the trivial one has alternatives and extensions. When we try to go through this system in a way of extensions without changing alternatives we will result in a deadlock (though theoretically we can expand infinitely but in non-computable manner).

Благоглупость (good stupidity). This thing is in principle very good. But in real world its small violations lead step by step to horrible consequences. Examples: tolerance, politcorrectness, communism, liberalism, democracy. This phenomenon can be revealed and studied by means of classical mathematics (non-stability of systems; divergence of effects in linear models and more deep second order models and so on).

Conceptual contradiction. Two things are non-contradictory but prevent development one another (poorly consistent). This effect was displayed in the theory of informalizable notions. It was observed but not recognized (maybe) first time in Programming. **Go to** and structured programming was classified by E. Dijkstra as mutually inconsistent ([17], 1968). Because there was no idea of informalizability at those times Dijkstra classified structured programming as good and **go to** as harmful. Later there was developed a method of automata programming alternative to structured programming and using explicit transition operators. Now there are a lot of examples of conceptual contradictions. For example such are “sanctity of human life”, “the right to life” and necessity of

death, finiteness of our life.

Classical mathematics is almost useless in searching and solving conceptual contradictions.

Sacred cow: it stands openly, obstructs many things, but nobody risks to see it and to remove it. For example the demand that all actions of programming language statements are to have the uniquely determined computational effect is a sacred cow. A whole herd of sacred cows was produced by formal equality, politeness and tolerance. We cannot study in which extents some people, religions or races are better or worse.

In informatics and mathematics (and in formal philosophy) the unique method to find sacred cows is now to try formalizing something *constructively*. Because these creatures hinder real actions, they cannot be placed into a constructive logic or theory.

2.6. Chaitin limit and programming languages

One of paradoxes arising while applying precise Computer Science to real Informatics can be solved from the point of view of Chaitin's limit. It is known that Kolmogoroff's complexity is invariant up to additive constant L . It follows then formally the complexity of programs in the different program languages is equivalent up to additive constant (Kolmogoroff theorem) and there is no reason to use different languages. Practice shows the opposite: program written by adequate tools can be 50 times shorter than in "universal" Java or C#. Why?

Theorem 1 states that $(k \cdot K_1) \leq (\text{CODE}[a] K_2) + C$ where k is equal to 1 if we consider standard programming codes. Constant C is a length of a translator program for the second language written in the first language. To write it eats almost all Chaitin's limit of a programmer.

Therefore we have an excellent and precise demagogic answer on a moronic and demagogic question very often posed to ones who did something by «exotic» language: "Is it possible to write the same in C# or Java?"

— Of course. It is possible to write all in the language of Turing machines, if you prefer.

Thus theoretical equivalence sometimes means practical incomparability.

This analogy works in other domains also. If we do not master a language of a concrete domain we can *in principle* to understand constructions and arguments but it is necessary to build in our mind a *translator* into our paradigm. Its complexity can be so high that it leaves almost no resources to analyze the argumentation.

Another warning. If you know many languages but have no background fundamental knowledge in your head you work worse than blind coder. Multi-tool method is effective only when a person masters a meta-knowledge, meta-method and a basis of notions.

So a fundamental knowledge is that which forms a system in a brain. Foundation of a system must be stable. It consists of a basis of relatively simple notions (keystones) amalgamated by a lot of relation and properties which show their interrelations gains, shortcomings and restrictions. It is ideal if in result a person sees restrictions of his/her system as a whole.

And there is one more bad side. Many people simply cannot appreciate complex (algorithmic) constructions such as recursions and even loops. They have no universal algorithm in their head. Here Chaitin's limit is 0 and this person simply cannot see something.

Final remark

It is false that clever one works faster than more stupid one. A stupid person never can understand what does a clever one and never can make the same work.

2.7. Benevolence to Other's Views

A problem of co-existence of different views is madly contaminated by "tolerance" originated in the fundamental mistake of J. S. Mill: he declared freedom of opinions instead of freedom of argumentation. He simply could not imagine that every irresponsible and moronic cry will demand rights and honors because it is an "opinion of a free person".

This goes deeper to Благоразумность of Voltaire: "I hate your opinions, but I would die to defend your right to express them". We see that there are too much people who accept no counter-

arguments against their opinions but are ready to kill each who criticize them. We see that there are too much people and institutions which substantiate their opinions not by argumentation but by direct lie, violence and manipulations (e.g. neo-liberals, neo-cons, fundamentalists, juvenile justice...)

Principle of benevolence to other's views.

Remember that The Truth is inaccessible to you and to any other human. So say confronting other views.

I do not agree with your views but you argue in their favor honestly and earnestly. I will defend your right to proclaim them, to substantiate them and to distribute them. In the same time I declare full and unrestricted right of me and of any other person to criticize them, to find weak points in your argumentations and maybe lie and manipulations.

This obligation is ended when your sights become refuted or you are caught on lie or manipulations (sophistic or psychological).

In the first case you remain an honest person for me and I will defend you against any attempts to punish you for error itself (but not for its consequences). If you will be so brave to recognize you have been mistaken I will help you to correct it and its consequences and you will become greater in my eyes.

If you would be caught on dishonored tricks all my responsibility would end. I will support the toughest of possible legal punishments for you because spiritual poison is more mortal than material.

3. Methodological Approach to Theology

3.1. Why Theology Can Become Applied Science and Other Reasons for New Approach

Informatics³ is a very unusual topic in a human kind activity. We found ourselves that we are able to create whole worlds by power of our mind and imagination (fantasy, ideas...). Computer plays here a role like to one of tongue and larynx in pronouncing our thoughts. So traditional engineering oriented towards material implementation is not very applicable in this domain.

Where worlds created by power of mind and will was considered earlier? This appears only in theology. So we are forced to return to theology enriched by experience of dreadful worlds created by ravings and ignorance of their architects and implementers.

If we consider Genesis as an example of programming of a complex system (our World) we can see that this was a well organized top-down process beginning from abstract objects (light and darkness) and finished by transfer this system to end user. This system was attempted to have a maximally friendly interface (the Paradise). But as usually it was invaded by a tester (or hacker) leading to temptation of Eve and to a critical error. Then system was debugged by adding functionality of death, making interface less friendly but more stable and even by full re-engineering (the Deluge).

Italo Calvino pointed [13] that initial project (as described by Hebrew priests) was conceptually contradictory and wicked. Immortal and innocent creatures having no notion of Good and Evil will occupy the entire world and behave extremely brutally counting only with rational and aesthetical arguments. They will have their felicity as the unique goal and the unique value.

This analysis once more proves that authors of Holy Bible were human beings but not God. They tried to understand His ideas and really appreciated them on very high level w.r.t. their time. So they described real HUMAN construction and implementation of very complex system. So people speaking that Holy Bible is God inspired are more precise than they would be. Its main ideas are inspired by God but were understood by restricted mind of humans and moreover after that transferred through several "disseminators" vulgarizing and distorting them.

There are other conceptions of worlds in different religions. Buddhist and Jainist worlds are natural ones, lawfully arising and collapsing. Jains also explicitly stated that there are many different worlds (some branches of Hinduism also).

But I know no religion in which the following is taken into account.

Any implementation of “In the beginning was the Word” (John 1:1) is partial and imprecise (because the Word is implemented in the unideal matter). Considering perfect creation of necessarily imperfect worlds leads us to a conception of perfect collection of worlds each of them is the best in some extents and their collection does not miss any positive property. Because usually a positive property is accompanied by a negative one (good by evil and so on) those worlds are alternatives covering the space of all best realizations. They give all possible ways to develop capabilities of souls and of beings as their implementations.

3.2. Methodological Argument for Deism

Chaitin's theorem showed that Kant was right stating that our intellect cannot solve a problem of God's existence.

So we have the following consequences.

1. Existence of God is a pseudoproblem from scientific point of view and you must take your own decision here.
2. It is unacceptable to cry that science rejects God (and equally that science proves God's existence).
3. It is inadmissible to make any scientific consequences from existence or non-existence of God.
4. It is acceptable to analyze this problem methodologically.

So, the problem of deism or atheism is a *methodological* problem. Stating a rational definition of God as *The Truth*, as *the unified highest Law of both nature and spirit which is beyond all worlds and all times*, as *the single Will which creates all laws and their realizations*⁴ we are inspired to find unity in difference, high level unifying notions and principles for realizations which seem to be not connected for plain thinking, or even contradictory for it, though both existing. It inspires us to develop ourselves both intellectually and spiritually and to keep these different sides and our material being in harmony. It inspires us to recheck our “precise and fully proved” results when applying them to real life.

In contrary atheism motivates us to idolize and adore our imperfect plain reasoning and our restricted knowledge and not to see ideal unifying beyond specialized “quasi-truths”.

This is a reason why a deist can easily be a non-religious person while atheists almost inevitably degrade to a fanatic quasi-religion. There is a simple criterion to recognize atheistic fundamentalist. *If a person begins to squirm and spew invectives seeing religious people or hearing a name of God he/she is really not an atheist but a Devil adorer.*

3.3. God as an ideal notion

Any description of a complex system begins to grow, to lose a form and in result to dilapidate if we use only “necessary entities”. To describe something meaningful and non-trivial is possible only through *ideal notions* (term of D. Hilbert). For example real numbers arose as idealization of physical measurement processes and different well coordinated scales of different devices. *Principle of Least Action* results in many particular principles and algorithms in different domains often very far from Newton mechanics in which it was discovered by P. de Maupertuis.

When ideal high level notions are used length of proofs and length of expressions shortens drastically. This effect can compress our calculation and speculation in tower of exponents times (as mentioned in section 2.3.) Moreover introducing ideal notions can often open new possibilities (Hao Wang).

When level of notions increased that what seems earlier completely unconnected surprisingly but naturally becomes different realizations and concretizations of single abstract ideal notion. For example logical conjunction, direct product in algebra, lower bound in lattices, and data record in

programming is all realizations of the same categorical construction. Main criterion whether unification is possible is a structural proximity but not a «background concrete stuff». The higher is level of notions the higher is level of their demands to harmony and the higher is their conformity. Roughly speaking we can pay almost no attention to conformity of knowledge and rules on level of direct pragmatic recipes and direct generalization of empirical facts (Horn formulas). It suffices that are not contradictory in small number of steps. But on higher levels each small conceptual inaccuracy results in a big problem (or in a swarm of “light” problems solving any of them we get at least two others⁵).

Thus practice step by step forces a human creator to conclude: ***there is no freedom of creation***. If you feign something (or somethink) this is a delirium. If you sorely master to see High Ideas and after that tormenting mastered to develop their adequate realization as precise and perfect as you could — this is a real creation. Life and happiness cannot be easy. Easy are existence and cheap enjoyment.

We result in single unifying idea, containing all common which is in its different realizations as harmony, knowledge, good, light: God.

After that we clarify for ourselves three derivative ideas. **First:** it is wrong to think on God as on the highest being. He is out of all worlds and all times. If we lower Him down to being we supplant Him by an idol of good lord. **Second:** the question whether God exists is a pseudo-question. **Third:** problem of God is not ontological but methodological in its essence. It is why the brilliant minds of Leibnitz, Newton, Schrödinger, Einstein, Spinoza, Lyubischev, and Pavlov resulted in deism, in firm assurance in idea and notion of God.

There are scientific “arguments” both for existence and for non-existence of God. Each of them becomes demagogic and sophistic after accurate critical analysis. The unique fair decision for a scientist is to recognize that this question cannot be solved rationally and this is a choice of a thinking person. Nobody can lay his burden of responsibility here to “objective scientific truth” or to religious authorities. Each human is to make this hard choice personally.

And one more ruthless consequence. God is beyond all religions and each religion claiming a monopoly in access to Him is a blasphemy⁶. Atheist stating: “I have no need in the hypothesis of God” or “There is a hard choice for each human which cannot be substantiated and made rationally/ I made my choice” is more honest and clean person than those who replaces faith by rituals and customs.

At all times a human stepped into in a great sin of pride. That people who first (or second after Ikhnaton and a handful of Egyptians) understood that nobody under sky is worth to worship (idols, human mind, human wishes which are worshiped by many atheists) immediately claimed itself as chosen one. All Abrahamic religions are restricting themselves by one small piece of one of worlds and by one of times treating it as the single existing. This is rejected by modern physics. They treated a human as a crown of creation which contradicts to biology, ecology and ethology. This is the same pride as “the chosen people” though extended on a bit larger area. Thus all these religions are really based on an original sin⁷.

Religions where this pride was rejected (Buddhism and Jainism) absolutize first-level knowledge which without an ideal notion in its background leads to emptiness (nirvana). They consider life as an encumbrance and a decline not as a gift and a value. It is necessary to remember that in initial Buddhism and Jainism higher beings are not gods or deities but teachers. They can show the way but it is meaninglessly to pray to them, to praise them and to ask their help because even the question “Does Buddha exists after para-nirvana?” is a quasiquestion.

3.4. Godly Inspired Ideas and Their Realizations: a Connection with Platonism

Mathematical and informatical practice shows us that high level ideas can be applied only after their concretization. The higher is level of an idea the higher (and incomparable stronger) is its potential might, the wider and more heterogeneous is its scope of application. This scope seems to be unlimited but an attempt to consider it as unlimited, aversion to understanding a person's knowledge limits in a moment kills its positive effect and leads to discrediting the idea itself. See as

example writings of genial physicist R. Penrose on informatics [19]. In the same time high level idea loses an attractive and useful property of empirical and direct recipes: possibility to apply immediately. The higher is a level of an idea the more steps are needed to realize it in a concrete circumstances and the higher is the effect of its application if this severities are overcome.

Say many people are frustrated if a recipe is equivalent to a Horn formula

$\forall x(A_1(x) \& \dots \& A_n(x) \Rightarrow B(x))$

They demand: "Say me directly what to do and not puff my brains by your *ifs*".

So a realization of a high order idea generates low level ideas which are easily applicable, more intelligible by majority but not adequate when a situation is changed. This is a reason why vote of majority is mistaken in any complex situation: they "understand" not ideas but their vulgarizations and common vulgarizations do not work here.

Now we reconsider Platonic view that things are realizations of highest and absolute Ideas. This view is in some sense the same as in the Gospel of John: in the beginning there was a Word or more precisely, an Idea of our world. This Idea goes directly from God. But each high idea is implemented through a chain of concretizations. Each concretization is not absolute and highest but they become more materialized and understandable. *Realization of an absolute Idea cannot be ideal.* This process become (objectively) beyond frame of Plato considerations.

This led us to tough but inevitable conclusion.

Highest absolute Platonic Ideas are not accessible by a human, Even mathematical notions are their incomplete, one-party, simplified and unideal realizations.

This conclusion is supported in high grade by Chaitin's theorem on incognizable. All things exceeding some limit of complexity are perceived as random and absolutely systemless. Nevertheless we can get an imagination that there is something beyond Chaitin's limit if we master several ideas and essences near to our limit. A common harmony which is existent in them and which cannot be explained and understood intellectually and rationally shows existence of more high essences beyond limits of our plain reasoning. I can add my and some other scientists' experience of introspection. In the state of divine inspiration (creative ecstasy, Samadhi fire) a person can see elusive outlines of much higher entities and can understand that they also are not absolute Ideas and there is something higher beyond them. This happens only if a person is not a fanatic of one idea and one method (unfortunately in the most cases it is so). This is why religious inspiration usually leads to absolutization and further to idolization of found issues and ideas. But scientific inspiration does the same too often...

Moreover non-classical mathematic showed that even objects which were considered as absolute (for example numbers) arise as a realization of the general idea in context of a couple of implicit suppositions. One of them for real numbers is an abstraction that our computations and measurements in principle can be absolutely precise. Moreover, last decade investigations show that there is one more dimension: main value and main resource. They are almost unknown to Western society because all attempts to publish them in Western journals broke due to ideological censorship: it is known that linear logic which is the logic of money is the logic for all resources. Last year a book containing main results on constructive logics was published in Germany but in Russian language [19].

Logics of static fully knowledgeable world (classical logic), pure knowledge (intuitionist logic), money (linear logic), time (nilpotent logic) and soul (reversive logic) are very different from the beginning and mutually inconsistent.

So it is a mortal trick to accept that persons with logic of money can develop society, science and so on.

All above considerations lead us to the conclusion:

Each realization of the Idea must have alternatives.

Let us continue to conclude. Each ideology, religion or theory which claims its own truth is wrong and leads to death. If there are questions which are "not to be discussed" that society is in a state of cruel disease. But alternatives cannot be discovered easily. *Negativism is one of forms of conformism.* To find an alternative a person is to be orthogonal to common views and prejudices.

This is a highest form of non-conformism where there can be issues conforming to general views of common society; there can be issues simply out of comprehension of majority; but the main distinction of this person is that he/her has a conceptually consistent system of knowledge and values in contrary to mosaic and chaotic common one.

Questions arise due to possible “regressus ad infiniti” when searching the highest Ideas. They can be removed by a simple analogy. Even in programming and informatics it is often more efficient to estimate finite entities (say number of steps in recursion) by infinite ordinals because they lead to result through a finite step of concretizations (calls).

3.5. Informalizability and God

The negative theology is considered by mature Orthodox Christianity as the most adequate for analysis of God. It states what cannot be God and what is not peculiar to Him. It refrained from positive statements about God. For example we can substantiate precisely that God is not a being, that He is not submitted to laws of Physics. We cannot state precisely whether He obeys logical laws and so we are to keep silence here. In contrary, to say that God is omnipotent is more like to unworthy flattery. Moreover notion of omnipotence itself is logically contradictory. Attempts to avoid this contradiction (say universal Turing machine) lead with necessity to possibility of failure this “omnipotent” device.

Because logic itself is also not absolute any attempts to describe God inside of classical logic (as in [20]) leads to substituting a term instead of the notion. The term “God” is rejected from the very beginning in our approach.

Thus any positive assertion considering God which is not derivable from negative ones is a hypothetical as almost all statements in [20]. Say we cannot assert that God is omniscience and Boolean understanding of “omniscience” is contradictory: non-classical leads to possibility of failures. But we can assert that God is infinite essence because assumption that He is finite can be easily refuted by *reductio ad absurdum*. Therefore we can accept a theorem of Nicolas Cusanus that trinity is not a contradictory notion [21]. So precise results in theology are possible but there are a small number of them and all they are to be examined carefully.

This is a reason why Spinoza rational definition of God as the substance with infinite number of infinitely perfect attributes remains the best one.

In the relation to God informalizability acts very ruthlessly. Each attempt to formulate precisely how to understand God and how to serve Him very easily leads to prejudices, worshiping of rituals or a book instead of God, fundamentalism, fanaticism and so on.

True religions in their best parts have some cures for these diseases. For example Islam theologians treat different branches of Islamic theology which are formally mutually contradicting as equally faithful because no human can understand Allah completely. Analogous but less clearly stated situation is in Judaism.

Therefore dogmatic theology can be useful first of all by its results which are independent from concrete dogmas. But there is one more possibility of its application. Because theology considered our world as an artificial object created by Mind of God and governed by Him it can be very useful in informatics because its models are much more elaborated and conceptually perfect than recipes of programmers. For example Christianity treats world as a program in beta-testing stage: this program works independently from Creator; there are some powerful testers the main of them is Devil and others include imperfect, arrogant and chaotically acting humans; Creator very rarely makes miracles to correct founded by testers bugs and in the perspective we see a full re-engineering of the whole system (Last Judgment). Islam treats the world as being under step-by-step debugging by very active Supervisor: in some branches of Islamic theology Allah re-creates the world at every moment; this is an excellent attempt to solve a contradiction of free will of human and full divine predestination.

Impossibility to prove rationally existence of God often leads people idolizing their poor mind and “rational thinking” directly into the embraces of Prince of this world. So, faith completely cleaned from fanaticism is for a scientist an excellent complement and a powerful tool of self-

testing.

And as a last remark. I wish to remember that in the theory of informalizable notions was proved that it is unacceptable to deduce precise and real corollaries from quasi-questions. So it is unacceptable to argue in a scientific work based both on existence and on non-existence of God. We cannot directly use in science neither deism nor atheism. And atheism in the form «I need no hypothesis of God» behaves so. It is correct as a scientific ideology and is much more correct in any aspect than religious or atheistic fundamentalism. But this does not prevent us to analyze methodologically whether this ideology is effective.

References

1. Nepejvoda N. N. *Chaitin's theorem as a source of methodological consequences*. 2012 ISBN978-5-288-05315-3, pp. 212–222.
2. Nepejvoda N. N. *Abstract variants of Godel and Chaitin theorems*. Will be published in DAN RAS, 2012.
3. Chaitin, G. J. *Information-theoretic limitations of formal systems*, J. of the ACM **21**, pp. 403–424.
4. Kleene S. C. *Introduction to metamathematics*. Amazon, 2009, ISBN: 978-0923891572.
5. G. Chaitin. *Meta Math!: The Quest for Omega*. Pantheon Books, 2005.
6. G. Chaitin. *Mathematics, Complexity and Philosophy*, Editorial Midas, 2011.
7. G. Chaitin. *Randomness in arithmetics*. Scientific American, 1988, No 9, pp. 56–68.
8. Shira Kritchman and Ran Raz. *The Surprise Examination Paradox and the Second Incompleteness Theorem*, Notices of the AMS **57**, N11 (2011).
9. PANU RAATIKAINEN. *ON INTERPRETING CHAITIN'S INCOMPLETENESS THEOREM*. Journal of Philosophical Logic **27**: 569–586, 1998.
10. I. Kant. *The Critique of Pure Reason*. <http://www.gutenberg.org/ebooks/4280>.
11. A. A. Розов. Депортация (Конфедерация меганезия -1) <http://lib.rus.ec/b/122647>
12. M. Aschbacher, *The status of the classification of the finite simple groups*, Notices Amer. Math. Soc. **51** (2004), pp. 736—740.
13. Calvino, Italo. *Cosmicomics* (trans. William Weaver). London: Picador, 1993.
14. G. Chaitin. *The Search for the Perfect Language*. Dicta & Contradicta, 2009 (can be accessed by <http://www.umcs.maine.edu/~chaitin/pi.html>).
15. Benedictus de Spinoza. *Ethica*. http://la.wikisource.org/wiki/Ethica_-_Pars_prima_-_De_Deo.
16. Непейвода Н. Н. *Прикладная логика*. Новосибирск, 2000, 565 с.
17. E.W. Dijkstra *Go To Statement Considered Harmful*. *Communications of the ACM*, Vol. 11, No. 3, March 1968, pp. 147-148.
18. R. Penrose *The Emperor's New Mind: Concerning Computers, Minds, and The Laws of Physics* (1989, ISBN 0-14-014534-6).
19. Nepejvoda N. N. *Уроки конструктивизма (Lessons of constructivism)*. LAP, Saarbrücken, 2011. ISBN: 978-3-8443-5181-1 78p.
20. Thomas Aquinas. *Summa Theologica* <http://www.newadvent.org/summa/>
21. Nicolas Cusanus. *De docta ignorantia*. <http://my.pclink.com/~allchin/1814/retrial/cusa2.pdf>

Notes

1. In Russian there are two words for English “creativity”. “Креативность” (creativity) means invention something new only to be new without real values and goals. “Творчество” (creation) means creation of new and useful things. This is why “creative class” is appreciated by Russians as a collection of uppity, spiritually and really impotent egocentric persons.
2. And not formalized, in contrary to common prejudice.
3. The European name ‘informatics’ seems much more reliable here than American one ‘computing’ because information is not in all case numerical.
4. This definition does not contradict to definition of Spinoza: “Per Deum intelligo ens absolute infinitum hoc est substantiam constantem infinitis attributis quorum unumquodque æternam et infinitam essentiam exprimit.” ([15], Definition VI) and can be considered as its complement due to current needs of constructive science.
5. This is even treated as an “objective law” for complex informational systems. A moment when small bugs begin to breed is considered as a moment when full re-engineering of a system is needed otherwise it will slowly and grievously die.
6. **Warning.** This does not mean anti-religious views. True religions collected a huge luggage of useful spiritual and psychological practices. They have a colossal experience in recognizing and curing mental, spiritual and psychological corruptions. To throw away this experience is a teenager thinking and arrogance. To accept the experience of a light and mature religion and be integrated into it is pragmatically one of the best decisions for those who has no will and forces to pass a way marked by Kierkegaard. Who is able to do this is following by his/her way to his Destiny and is performing his/her Mission. He takes on his own breasts all negative consequences of his actions.
And the most terrible heresy in each religion and in each ideology is fanaticism and fundamentalism.
7. The original sin can have a rational background in our conception (which is independent from myth on Eve and an apple). Approximately a half of all information which a human processes during life this creature gets in the womb. Thus a newly-born child is infected by sins, prejudices, vices and often by diseases of parents (especially of mother). This leads to a tough consequence. Pregnancy is a honorable (not shameful) state. To train and to develop a child is necessary from the womb of mother and treat an embryo as a human being. Right to abortion is logically equivalent to right of parents to kill their child (this right existed, say, in Rome).

Intensio: Leibniz in Creating a New Term for the Modal Logic

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Abstract:

It is still difficult to say what the main source of Leibniz's modal thinking was; at least, his acquaintance with the ideas of Spanish Jesuits about the "moral necessity" is to be dated to the epoch when the modal ideas already took shape in his mind. There was, however, one name normally referred to by Leibniz himself as his main predecessor in modal thinking, Richard Swineshead. In fact, Leibniz created his personal myth about Swineshead even before having read his works, and so, he attributed to Swineshead some of his own ideas, including the modal reinterpretation of the term *intensio* borrowed from the mediaeval physics.

Leibniz's achievements and intuitions in the field of intensional logics were evaluated, for the first time, by no other than the creator of the modern modal logic Clarence I. Lewis, whose seminal 1918 monograph contains a very important historical essay on Leibniz with addition of two translations of his pertinent works (published for the first time in 1903, but not acknowledged as important even then). [1, pp. 5-18, 373-387] Then, Leibniz's ideas about intensionality were studied in a more systematic way by Nicholas Rescher, [2] another key figure in the twentieth-century modal logic.

It is still a disputable matter, whether Leibniz had direct predecessors in his modal thinking. It is often thought that, in the matters of theodicy, he had ones – Spanish Jesuit thinkers of the seventeenth century who were teaching about the "moral necessity" for God and even the "possible worlds." [3] It is certain that Leibniz did have access to their publications, although was not referring to them explicitly. However, Bartholomew Des Bosses, another Jesuit and a correspondent of Leibniz, who was the first to notice the parallels between these Jesuits' and Leibniz's thought, did not attribute to them any direct influence on Leibniz. [4, pp. 228/229 (lat./Eng. tr.) and 438, n. 5.]¹

The German mystical thought of Weigel, F. M. van Helmont, and Böhme could also be a source of inspiration for Leibniz's modal thinking, but this possibility remains unexplored, and, anyway, Leibniz did not recall any of them in explicitly modal contexts.²

Normally, Leibniz presented his ideas concerning the modal logic as his original ones. There is, however, a unique name which is often referred to in Leibniz's works as his predecessor in modal thinking, Richard Swineshead. Moreover, Leibniz's modal term *intensio*, so popular in the modern logic, goes back to Leibniz's understanding of Swineshead. Probably, however, the modern historians of the modal logic had reason to pay little attention, if any, to Swineshead. Leibniz's admiration toward Swineshead is a phenomenon whose value is somewhat independent from the historical personality of Swineshead as a scholar.

Mary Spencer in her 1971 notice showed how the modern use of the term “intension” and its derivatives goes back to Leibniz. [5] Since then, some previously unpublished Leibniz’s papers became available,³ and their contents allow us to grasp Leibniz’s intuition in a more adequate way. The scholastic background of Leibniz’s usage of *intensio* has been noticed but never traced, and this is the main reason to readdress the issue after Mary Spencer.

In his earlier period, Leibniz knew the word *intensio* only in the sense of the late Scholasticism, where it was a physical term (roughly with the same meaning as the modern “intensity”)⁴ forming a pair with its antonym *remissio*. The fourteenth-century scholastic debate concerning “intensions” and “remissions” of forms was about physics. Leibniz, however, was thinking about physics in terms of semantics. Moreover, his way of thinking was influenced by the logic of Port Royal (1662) with its distinction between “extension” and “comprehension”⁵; it is rather obviously that the term “extension” in Leibniz’s usage goes back to Arnauld and Nicole.

Somewhere before 1681, Leibniz started to develop a very high idea, if not a myth, about his alleged predecessor in Scholastics, Richard Swineshead (*fl. ca.* 1340–1355) nicknamed Calculator,⁶ then known to Leibniz only indirectly from the references by other authors (only one of them is called by name: Scaliger⁷). In one instance, Leibniz said that, judging from the works of Swineshead’s followers (“*ejus sectatorum scripta*”), their merits in applying mathematics “in media metaphysicorum” (“in the field of metaphysics”) must be praised, and, probably, they would anticipate “our works,” were they reached by “the presently achieved light of mathematics” (“*lumen Mathematicorum quod nunc accensum est*”). [6, p. 720] Leibniz’s attitude toward both Swineshead and Scholasticism is clear from the following passage: “Parmy les Scholastiques il y eut un certain Jean Suisset appellé le Calculateur, dont je n’ay encor pû trouver les ouvrages, n’ayant vue que ceux de quelques sectateurs qu’il avoit. Ce Suisset a commencé de faire le Mathematicien dans le Scholastique, mais peu de gens l’ont imité, parce qu’il auroit fallu quitter la methode [des] disputes pour celle des comptes et raisonnemens, et un trait de plume auroit epargné beaucoup de clameurs.”⁸

According to Leibniz’s impression which was already formed as early as in 1682, Swineshead must be placed alongside with Aristotle!⁹ In other instances, Leibniz enumerates Swineshead’s studies among the most important achievements in philosophy.¹⁰ It is obvious that Leibniz, long before reading Swineshead, already considered him as the inventor of logical “calculus,” the main goal of Leibniz’s own studies. When, in December 1689, Leibniz eventually found Swineshead’s incunabula in Florence, he was very glad and, of course, did not change his opinion.¹¹ It was certainly a forcible interpretation of Swineshead’s legacy, but in our present situation of lacking detailed studies in Swineshead and even critical edition of his works¹² it would be hasty to judge in what extent Leibniz was indulging in wishful thinking.

The real Swineshead participated in the circle of British schoolmen which considered the qualities (“forms”) as able to change in intensity without being changed themselves (that is, remaining the same individual forms but differing in intensity).¹³ His main innovation in the field consisted in introducing a specific way of counting the quantity of a given form. He proposed to start from the zero grade (not from the maximum grade), and so, *de facto* to count only the “intension” (intensity), because the “remission” becomes an equivalent magnitude whose counting from the zero grade is inconvenient.¹⁴ This is basically the modern approach to measurement of physical magnitudes. Apparently, however, there is no sign that Swineshead himself applied his theory outside physics and considered it as a universal logical computus¹⁵—as his admirer Leibniz certainly did.

In one of the earliest notices mentioning *intensio*, Leibniz gives the following definitions: “*Intension* is the quantity of the form itself, such as if the form is motion, intension would be speed. *Extension* of a form is the quantity of matter which is within the form of the same measure, such as the quantity of the moving body is the extension of the motion.”¹⁶ These definitions are still in Swineshead’s vein. But even before reading Swineshead Leibniz started to use the notion intension for the logic of natural language, for the phenomenon which we now call indexicality. Thus, he

wrote: “In the pronouns, we have some intension, such as *ego, egomet; tu, tute; ille, illemet* or *ille ipse, ipsemet*.” [7, p. 888]¹⁷ This is not an intensional in the modern sense (such as in the Montague semantics), but simply a dimension of meaning. The indexicals, such as the pronouns, do not have a function which ascribes to them denotations in each of the possible worlds (as does the intensional in Montague’s sense).

Such was the background of the now famous Leibniz’s passage in the *Nouveaux Essais sur l’entendement humain*, IV, xvii, 8: “La maniere d’enoncer vulgaire regarde plustost les individus, mais celle d’Aristote a plus d’egard aux idées ou universaux.”¹⁸ Car disant ‘tout homme est animal’, je veux dire que tous les hommes sont compris dans tous les animaux; mais j’entends en même temps que l’idée de l’animal est comprise dans l’idée de l’homme. L’animal comprend plus d’individus que l’homme, mais l’homme comprend plus d’idées ou plus de formalités; l’un a plus d’exemples, l’autre plus de degrés de réalité; l’un a plus d’extension, l’autre plus d’intension.”[9, p. 486]¹⁹

Now Leibniz’s approach—which Leibniz himself considered as being Swineshead’s one—became called-for in the Quantum logics, where the physical phenomena are treated with the logical methods developed for the philosophy of language. Leibniz applied to the language the logical ideas inspired by Swineshead’s physics, but now the logicians of physics use logical ideas of the philosophy of language and, in general, of the modal logic, which go back to Leibniz.²⁰ In both cases, both physics and language are treated within some general semantic approach. The circle is closed.

Moreover, the modern Quantum logics are continuing Leibniz’s ideas of the last year of his life (1716), when he reconsidered, in the IV and V letters to Clarke, his own (now called Leibniz’s) principle of the identity of indiscernibles.[10] The violation of this principle in the world of Quantum phenomena is the main reason of the irreducible intensionality in the corresponding Quantum logics.²¹

Leibniz, as it seems, did not explain the reasons of his own predilection toward the intensional semantics; on the contrary, he always explained his intensional calculi in the extensional terms as well (calling such an extensional approach the “Scholastic” one [11, p. 200]²³. Were Leibniz continue his work after 1716, he would enface the serious asymmetry between the intensional and extensional semantics of the world, which would justify his (after Aristotle) choice of the intensional approach as the basic one.

References

1. Lewis 1918. Cf. Couturat 1903, and, before this, Couturat 1901.
2. Rescher 1954/2006. Cf. Swoyer 1995; Bradley 1998.
3. Cf. Knebel 2000; Schmutz 2006.
4. Leibniz 2007
5. Spencer 1971.
6. *Elementa Rationis* (1686); Leibniz 1999, p. 720.
7. Leibniz 1999
8. Cf. Matthews 1992
9. Leibniz 1990
10. Chernoff 1981
11. Leibniz 1999

Notes

1. Des Bosses 1719. One can see, from Leibniz's letter to Des Bosses dated to 15 February 1712, that it was Des Bosses himself who introduced Leibniz to this Jesuit doctrine of divine "moral necessity": Leibniz 2007. Such a late date precludes any possibility that this Jesuit doctrine was among the *sources* of Leibniz's modal thought.
2. As an introduction to this topics, s., most recently, Coudert 2011, and, among earlier publications, especially Coudert, Popkin, Weiner 1998.
3. Published for the first time in Leibniz 1999.
4. Cf. Leibniz's often quoted long philosophical letter to his disciple and friend Arnold Eckhard (1677), where he gives the following definitions: "...*perfectionem* esse gradum seu quantitatem realitatis seu essentiae, ut *intensio* gradus qualitatis, et *vis* gradus actionis" (Leibniz 2006, p. 543. Nr 148) — "...*perfection* is degree or quantity of reality or essence, as *intensity* is degree of quality, and *force* is degree of action" (Leibniz 1989, p. 177).
5. Cf. definition in Port Royal's *Logique, ou L'art de penser*, II, xvii: "...il faut distinguer dans les idées la comprehension de l'extension, & que la comprehension marque les attributs contenus dans l'idée, & l'extension, les sujets que* contiennent cette idée <* *variant reading*: ...sujets qui participent et contiennent cette idée selon sa comprehension>" (Arnauld, Nicole 1981, p. 169). Translation: "...in the ideas, one has to discern between the comprehension and the extension, in the way that the comprehension designs the attributes contained in an idea, and the extension the subjects which contain this idea <*variant reading*: ...subjects which participate and contain this idea according to the comprehension>."
6. There is no detailed study of him; cf. the most comprehensive article: Mudroch, Sylla 2008.
7. Sc., Julius Caesar Scaliger (1484–1508); his "Eloge" to Swineshead is mentioned by Leibniz in his letter to Antonio Alberti (20 January 1690): Leibniz 2009, p. 306. The editors provide (*ibid.*) the exact quote from Scaliger, which, probably, contributed to formation of some "cult" of Swineshead in Leibniz: "Joanni Suisset calculatori, qui pene modum excessit ingenii humani," that is, who "almost surpassed human abilities." Cf. also Leibniz's letter to Justel quoted in n. 16 below. There were three Swinesheads in the 14th-century Oxford, Richard, John, and Roger, and Leibniz during the whole his life attributed to "Calculator" Richard the name of the lawyer John, who left no works.
8. *Projet et essais pour avancer l'art d'inventer* (dated from August 1688 to October 1690, but the citation obviously predates December 1689, when Leibniz read Swineshead in Florence); Leibniz 1999, p. 965, cf. p. 945 for datation. Tr.: "Among the Schoolmen, there was a certain John Suisset named Calculator, whose works I was unable to find out so far, having seen only those of followers which he had. This Suisset started to do mathematics in scholastics, but few people imitated him, because (otherwise) one would have to abandon the method of disputes and (to take) instead the method of computations and reasoning, in the way that one stroke of pen would eliminate much screams."
9. *Ad Praefationem Elementorum veritatis aeternae* (1682): "Dicam nunc de illis qui Methodum demonstrativam ad Metaphysica et Moralia transtulere. Primus aliquid in hoc genere praestitit Aristoteles, cujus libri *Primorum Analyticorum* utique sunt demonstrativi, et scientiam conduunt circa materiam ab imaginatione remotam. Inter Scholasticos quidam Joh. Suisset, vulgo dictus calculator, Mathematicum aliquid affectavit, et de intensione ac remissione qualitatum solito subtilius ratiocinatus est"; Leibniz 1999, p. 446. Tr.: "Now I say about those who applied the method of [logical] demonstration to metaphysics and moral matters. Aristotle was the first who showed something in this genre, whose books *Prior Analytics* are certainly demonstrative and led scholarship in the matters remote from imagination. Among the Scholastics, certain John Suisset, nicknamed Calculator, explained something mathematically, and reasoned about intension and remission of qualities in more details than usually."
10. *Catalogus inventionum in logicis* (early 1681?); Leibniz 1999, p. 427 — this seems to be the earliest piece of the whole Swinshead's dossier in Leibniz; cf. *De arte characteristica ad perficiendas scientias ratione nitentes* (1688); Leibniz 1999, p. 910.
11. Letter to Antonio Alberti, 20 January 1690 (s. note 11), p. 306: "J'y ay vû aussi un livre imprimé vers la fin du 15[.] siècle [= either Padua, ca 1477 or Pavia, ca 1498] que j'avois désiré de voir il y a long temps, sçavoir Johannis Suisset *Calculationes de Motu, et intensionibus ac remissionibus formarum seu qualitatum*. Il estoit fameux sous le nom de Calculator. <...> C'estoit quelque chose de singulier, qu'un scholastique raisonnât Mathématiquement [tr.: I have seen, moreover, a book published in the late 15th century, which I was wishing to see since long time, namely, John Suisset's *Computations concerning Movement and Intensions and Remissions of Forms, that is, Qualities*. He was famous under the name Calculator]" letter to Henri Justel (29 July/8 August 1692): "J'avois cherché long temps les oeuvres du celebre Suisset, scholastique Anglois, dont Jules Cesar Scaliger et autres parlent avec grandissime eloge; il avoit introduit les Mathematiques dans la Scholastique; et on l'appelloit pour cela le Calculateur. Mais ses ouvrages sont devenus si rares, à cause de l'oubli sans doute, et du mépris qu'on a eu depuis pour ces études, que je ne les ay vûs qu'à Florence. <...> Cependant je remarquay qu'il y avoit des pensées profondes [tr.: I was looking since long time for the works of the famous Suisset, an English schoolman, about whom Julius Caesar Scaliger and others say with much praise; he introduced mathematics in scholastics, and was named, because of this, the Calculator. But his

- works became so rare—because of oblivion, I am sure, and outcast which he undergone after this because of these studies—that I had not seen them before Florence. <...> Nevertheless, I noticed that there were, here, some deep thoughts]” (Leibniz 2009, p. 555).
12. Critical edition of Swineshead’s *Calculator* was a long-life task of Marshall Clagett, which he left unfulfilled. Cf. Murdoch 2006, p. 17.
 13. The alternative viewpoint, elaborated by some British scholars, consisted in considering the changes of the “forms” as destruction of the previous “forms” and their substitution with the next ones. Thus, any degree of temperature, for example, was considered as a specific form which is to be destroyed when the temperature changes. Cf. Shapiro 1959.
 14. Cf. the only detailed study in the field, where all the relevant citations are quoted with the variant readings: Clagett 1950/1979; neither subsequent parts of this study nor critical edition of Swineshead were produced by the author.
 15. I consulted Swineshead in the most accessible edition, slightly different by contents from that which consulted Leibniz: Suiseth 1520.
 16. *Specimena de motus causa et de corporum qualitatibus* (between 1678 and 1681): *Intensio* est quantitas formae in se, ut si forma sit motus, intensio erit celeritas. *Extensio* formae est quantitas materiae cui inest forma homoeomerica, ut quantitas corporis moti est ipsius motus extensio; Leibniz 1999, p. 2016 (ed. princeps). This notice is roughly contemporary to the earliest mentions of Swineshead in Leibniz’s papers.
 17. *De lingua philosophica* (1687–1688): In pronomibus habemus quandam intensionem, ut *ego*, *egomet*; *tu*, *tute*; *ille*, *illemet* seu *ille ipse*, *ipsemet*;
 18. Leibniz was basically right in this understanding of Aristotle. [8, pp. 17-23]
 19. Tr.: “The common mode of statement regards rather individuals, but that of Aristotle ideas or universals. For in saying, every man is an animal, I mean to say that all men are included in all animals; but I mean at the same time that the idea of animal is included in the idea of man. Animal includes more individuals than man, but man includes more ideas or more formalities; the one has more examples, the other more degrees of reality; the one more extension, the other more intension” (Leibniz 1896, p. 569).
 20. Among the pioneering works one should name Goldblatt 1974 and Toraldo di Francia 1985. As Vasjukov noticed, Goldblatt uses, in his Kripkean frames, a function analogue to the intensionals of Montague: Vasjukov 2005, pp. 105-106. For a detailed discussion of designation and description problems, s. Dalla Chiara, Toraldo di Francia 1992. S., moreover, other most important works: Dalla Chiara 1987; Dalla Chiara, Krause 1998; Dalla Chiara, Giuntini, Greechie 2004, esp. pp. 199-200.
 21. Cf., beside the bibliography in n. 25, French, Krause 2006.
 22. *Elementa calculi* (1679): In scholis aliter loquuntur, non notiones spectando, sed exempla notionibus universalibus subjecta [tr.: In scholastics, it is said differently, not in the aspect of notions but the individuals put under universal notions]

Literature

1. Arnould, Nicole 1981 — Arnould, Antoine et Nicole, Pierre. *La logique ou L’art de penser: contenant, outre les règles communes, plusieurs observations nouvelles, propres à former le jugement*. Éd. critique par Pierre Claire, Pierre et François Girbal. Bibliothèque des Textes Philosophiques. Paris: Vrin, 1981.
2. Bradley 1998 — Bradley, Brassler O. “Leibniz on Intension, Extension, and the Representation of Syllogistic Inference.” *Synthese* 116 (1998): 117-139.
3. Chernoff 1981 — Chernoff, Fred. “Leibniz’s Principle of Identity of Indiscernibles.” *The Philosophical Quarterly* 31 (1981): 126-138.
4. Clagett 1950/1979 — Clagett, Marshall. “Richard Swineshead and Late Medieval Physics: I. The Intension and Remission of Qualities (1).” *Osiris* 9 (1950): 131-161; repr. in: idem, *Studies in Medieval Physics and Mathematics*. Variorum Collected Studies Series CS 103; London: Variorum, 1979, ch. VIII.
5. Coudert 2011 — Coudert, Allison P., *Religion, Magic, and Science in Early Modern Europe and America*. Praeger Series on the Early Modern World. Santa Barbara, CA, etc.: Praeger, 2011.
6. Coudert, Popkin, Weiner 1998 — Coudert, Allison P., R. H. Popkin, G. M. Weiner (eds.), *Leibniz, Mysticism and Religion*. Archives internationales d’histoire des idées 158. Dordrecht: Kluwer Academic Publishers, 1998.

7. Couturat 1901 — Couturat, Louis, *La logique de Leibniz d'après des documents inédits*. Collection historique des grands philosophes; Paris: Alcan, 1901.
8. Couturat 1903 — Couturat, Louis, *Opuscules et fragments inédits de Leibniz. Extraits des manuscrits de la Bibliothèque royale de Hanovre*. Paris: Alcan, 1903.
9. Dalla Chiara 1987 — Dalla Chiara, Maria Luisa. "An Approach to Intensional Semantics." *Synthese* 73 (1987): 479-496.
10. Dalla Chiara, Giuntini, Greechie 2004 — Dalla Chiara, Maria Luisa, Giuntini, Roberto, Greechie, Richard. *Reasoning in Quantum Theory. Sharp and Unsharp Quantum Logics*. Trends in Logic. Studia Logica Library. Dordrecht: Kluwer Academic Publishers, 2004.
11. Dalla Chiara, Krause 1998 — Dalla Chiara, Maria Luisa, Krause, Décio. "Quasiset Theories for Microobjects: a Comparison," in: Castellani, Elena (ed.) *Interpreting Bodies: Classical and Quantum Objects in Modern Physics*. Princeton: Princeton UP, 1998, pp. 142-152.
12. Dalla Chiara, Toraldo di Francia 1992 — Dalla Chiara, Maria Luisa, and Toraldo di Francia, Giuliano. "Individuals, Kinds, and Names in Physics," in: Corsi, Giovanna, Dalla Chiara, Maria Luisa, and Ghirardi, Gian Carlo (eds.) *Bridging the Gap: Philosophy, Mathematics, and Physics*. Boston Studies in the Philosophy of Science 140. Dordrecht: Kluwer Academic Publishers, 1992, pp. 261-283.
13. Des Bosses 1719 — Des Bosses, Bartholomeus, "Monitum interpretis," in: Leibnitius, G. G., *Tentamina Theodicaeae de bonitate Dei, libertate hominis et origini mali*, T. I, Francofurti: C. J. Bencard, 1719 [separate foliation].
14. French, Krause 2006 — French, Steven, Krause, Décio. *Identity in Physics: A Historical, Philosophical, and Formal Analysis*. Oxford: Clarendon Press, 2006.
15. Goldblatt 1974 — Goldblatt, Robert I. "Semantic Analysis of Orthologic." *Journal of Philosophical Logic* 3 (1974): 19-35.
16. Knebel 2000 — Knebel, Sven K., *Wille, Würfel, und Wahrscheinlichkeit. Das System der moralischen Notwendigkeit in der Jesuitenscholastik, 1550–1700*. Paradeigmata 21; Hamburg: Meiner, 2000.
17. Leibniz 1896 — Leibniz, Gottfried Wilhelm. *New Essays concerning Human Understanding*. Tr. A. G. Langley. London: McMillan & Co., 1896.
18. Leibniz 1989 — Leibniz, Gottfried Wilhelm. *Philosophical Papers and Letters. A Selection Translated and Edited, with an Introduction by Leroy E. Loemker*. The New Synthese Historical Library. Texts and Studies in the History of Philosophy 2. Dordrecht etc.: Kluwer Academic Publishers, ²1989.
19. Leibniz 1990 — SSBAA, Reihe VI: *Philosophische Schriften*, Bd 6: Robinet, André, and Schepers, Heinrich (eds.) *Nouveaux Essais*. Berlin: Akademie Verlag, ²1990 [repr. 2006].
20. Leibniz 1999 — SSBAA, Reihe VI: *Philosophische Schriften*, Bd 4: Schepers, Heinrich et al. (eds.) *1677 – Juni 1690*. Berlin: Akademie Verlag, 1999.
21. Leibniz 2006 — SSBAA, Reihe II: *Philosophischer Briefwechsel*, Bd 1: Hochstetter, Erich et al. (eds.) *1663–1685*. Berlin: Akademie Verlag, ²2006.
22. Leibniz 2007 — Leibniz, Gottfried Wilhelm. *The Leibniz–Des Bosses Correspondence*. Translated, Edited and with an Introduction by Brandon C. Look and Donald Rutherford. The Yale Leibniz; New Haven—London: Yale UP, 2007.
23. Leibniz 2009 — SSBAA, Reihe II: *Philosophischer Briefwechsel*, Bd 2: Schneider, Martin, et al. (eds.) *1686–1694*. Berlin: Akademie Verlag, 2009.
24. Lewis 1918 — Lewis, Clarence Irving. *A Survey of Symbolic Logic*. Berkeley: University of California Press, 1918.
25. Matthews 1992 — Matthews, Gareth B. "Container Metaphysics According to Aristotle's Greek Commentators," in: Bosley, Richard, and Tweedale, Martin (eds.) *Aristotle and His Medieval Interpreters*. Canadian Journal of Philosophy Supplementary vol. 17; Calgary: University of Calgary Press, 1992, pp. 17-23.
26. Murdoch 2006 — Murdoch, John. "In memoriam. Marshall Clagett (1916–2005)." *Aestimatio* 3 (2006): 14-18.

27. Mudroch, Sylla 2008 — Mudroch, John E., Sylla, Edith Dudley. “Swineshead (Swyneshead, Suicet, etc.), Richard,” in: *Complete Dictionary of Scientific Biography*. 2008. *Encyclopedia.com*. (May 20, 2012). <http://www.encyclopedia.com/doc/1G2-2830904227.html>.
28. Rescher 1954/2006 — Rescher, Nicholas. “Leibniz’s Interpretation of His Logical Calculi.” *The Journal of Symbolic Logic* 19 (1954): 1-13; repr. in: idem, *Collected Papers*. Vol. X: *Studies in the History of Logic*. Heusenstamm: Ontos Verlag, 2006, pp. 141-151.
29. Schmutz 2006 — Schmutz, Jacob, “Qui a inventé les mondes possibles ?” in: J.-C. Bardout, V. Jullien (eds.), *Les mondes possibles*. Cahiers de philosophie de l’Université de Caen 42; Caen: Presses universitaires de Caen, 2006, pp. 9-45.
30. Shapiro 1959 — Shapiro, Herman. “Walter Burley and the Intension and Remission of Forms.” *Speculum* 34 (1959): 413-427.
31. Spencer 1971 — Spencer, Mary. “Why the ‘S’ in ‘Intension’?” *Mind* N.S. 80 (1971): 114-115.
32. SSBAA — Leibniz, Gottfried Wilhelm. *Sämtliche Schriften und Briefe (Akademie Ausgabe)*.
33. Suiseth 1520 — Ricardus Suiseth Anglicus. *Calculator*. Venice, 1520.
34. Swoyer 1995 — Swoyer, Chris. “Leibniz on Intension and Extension.” *Noûs* 29 (1995): 96-114.
35. Toraldo di Francia 1985 — Toraldo di Francia, Giuliano. “Connotation and Denotation in Microphysics,” in: Mittelstaedt, P., and Stachow, E. W. (eds.) *Recent Developments in Quantum Logics*. Mannheim: Bibliographisches Institut, 1985, pp. 203-214.
36. Vasjukov 2005 — Васюков, В.Л. *Квантовая логика* [Vasjukov, V.L. *Quantum Logic*]. Moscow: PerSe, 2005.

On Axiomatization of Non-Cartesian Logics¹

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Abstract:

The paper analyses a non-Cartesian logic Sm_4 as a typical sample of rather a wide class of logics which have unseparable pairs of truth-values in their minimal matrices. The algorithm for construction of bivalent semantics, described by Caleiro et al., cannot be directly applied to these logics.

Keywords:

non-Cartesian logic, logical matrix, unseparable truth-values, axiomatization

Nowadays it is generally accepted as a kind of truism by a significant part of logicians that Belnap's four-valued logic B_4 (the definition see below) is a good logical system, which is both useful in practice and fruitful in theory. Lots of papers and monographs deal with the syntactic analogue of B_4 , which is a well-known system of First Degree Entailment (FDE), and with its algebraic correlate, id est the class of De Morgan algebras.

A quite disappointing conclusion that one may derive from results of this huge research in the mentioned field is that B_4 is something very much like a piece of hard concrete, not allowing any constructive modification without changing its nature to the extent which cannot satisfy any final user, neither a philosopher, nor a logician.

This observation seems to be plausible in view of the fact that after all manipulations with B_4 the structure of logic itself remains unchanged, if we treat B_4 as the power set of the set of classical truth-values True and False and proceed with taking power sets of our resulting sets, we can, after all, only obtain the same system, characteristic for FDE (see [5]).

In this paper the author considers possible ways, which, as it seems to him, may be interesting in their philosophical and technical implications, of modification for B_4 and other logical matrices, characteristic for FDE. The basic fact we use is the concept of logical consequence in B_4 disguises the existence of distinguished values in matrices, characteristic for this logic. Once we start trying to deal with B_4 as the logic which really has designated values, we can obtain some new logics just with changing the set of designated values of the original logic and, maybe, slightly modifying definitions of logical connectives in matrices, characteristic for FDE, or adding new connectives to them.

At first, we need the well-known definition of Belnap's connectives with following truth-tables:

$\&$	t	b	n	f
t	t	b	n	f
b	b	b	f	f
n	n	f	n	f
f	f	f	f	f

$\sim$	x
f	t
b	b
n	n
t	f

Using these definitions of conjunction and negation, we can also define disjunction in the usual way, just putting $A \vee B =_{\text{def}} \sim (\sim A \& \sim B)$. The definition of logical consequence is not canonical, because it does not use the notion of designated values and preserving them from premises to conclusion:

$\Gamma \models A$, if and only if for every interpretation of formulae γ in Γ the value of γ is less or equal to the associated value of A with respect to the partial order on the set of truth-values: $f \leq b \leq t$ and $f \leq n \leq t$.

Shramko and Zaytsev, however, in [4] proved that using this definition of logical consequence is equivalent to using a canonical one, putting the set of designated values as $\{t, b\}$. But what happens, if we do not want the definitions to be equivalent, if we do not think that changing always means spoiling? In this case one may consider a new logic with Belnap's connectives and a single designated value $\{t\}$. With present definitions this leads to reconstruction of all paradoxes of classical logical consequence. Still, with some modifications we can obtain a logic, which is a kind of brand-new.

Until now we only considered formulae, which do not involve an implication-style connective. There is a possibility of adding the implication of Smiley to the set of Belnap's connectives. Smiley's implication $A \rightarrow B$ gives the value t, if and only if the value associated to A is less or equal to the value associated to B ; it gives the value f in all other cases. Thus defined connective has the following truth-table (for more information on Smiley's implication see [2]):

$\rightarrow$	t	b	n	f
t	t	f	f	f
b	t	t	f	f
n	t	f	t	f
f	t	t	t	t

In addition, we change the definition of negation on the values b and n, now $\sim b = n$ and $\sim n = b$. Thus, the new truth-table is:

$\sim$	x
f	t
n	b
b	n
t	f

The resulting matrix for this new logic is the following:

$Sm_4 = \langle \{t, b, n, f\}, \{t\}, \{\sim, \&, \rightarrow\} \rangle$, where $\{t, b, n, f\}$ is the set of truth-values; $\{t\}$ is a set of designated values; $\{\sim, \&, \rightarrow\}$ is the set of modified logical connectives.

We define logical consequence in the usual way in terms of preserving “truth” from premises to conclusion:

$\Gamma \models A$, if and only if A has the value t , whenever all γ in Γ have the value t .

In contrast with B_4 , this new logic Sm_4 is neither relevant, nor paraconsistent any more. Still, its matrix among most of the logical systems described in literature enjoys rather a rare property, which the author of this paper in [3] has called “being non-Cartesian”. Non-Cartesian logic is a logic which has at least one pair of unseparable truth-values in its least by cardinality characteristic matrix. The notion of separability for truth-values is used by Caleiro et al. in [1] in their algorithm of constructing bivalent semantics for many-valued logics. Most logics have enough linguistic expressive power to make every pair of truth-values in their minimal characteristic matrix separable. Such logics (id est the predominant part of all finitely-valued logics) can be called Cartesian. The definition of a separable pair of truth-values v_1 and v_2 is the following:

Truth-values v_1 and v_2 are called separable, if and only if

v_1 is in the set of designated values, if and only if v_2 is not in this set; or

it is possible to find a formula in the language of the logical system in question such, that this formula only contains a single propositional variable p_i and logical connectives, and the truth-value, assigned to this formula under the interpretation of p_i with one of the values v_1 or v_2 , is in the set of designated truth-values, if and only if the truth-value, associated to this formula under the interpretation of p_i with the other truth-value from the pair v_1 and v_2 , is not in the set of designated values.

So, if a logic has this separability property for every pair v_1 and v_2 of truth-values in its minimal characteristic matrix, it can be called Cartesian, otherwise it is non-Cartesian. One can easily check that the formulated logic Sm_4 is non-Cartesian, as the truth-values b and n in its matrix, which indeed is a minimal one, cannot be separated using any formula, constructed just with a single propositional atom and any composition of the connectives from the set $\{\sim, \&, \rightarrow\}$. This logic validates all of the axioms and rules of the relevant system E (of entailment), but fails to validate the specific axiom of system R . Therefore, it can be dealt with as an explosive extension of E .

Every logic which has a “Cartesian” minimal characteristic matrix can always be endowed with a “non-Cartesian” characteristic matrix, which cardinality is not minimal, but in such cases it is, obviously, possible and rather easy to get rid of the excessive truth-values. In case of truly non-Cartesian logics, one cannot just throw away any of the elements of non-separable pairs without changing the logic itself. On the other hand, it is possible to add some operators to the language of a non-Cartesian logic to make it Cartesian. In particular, it is enough (if possible) to add all functions $J_i(x)$, where i is an element of the set of truth-values, and $J_i(x) = 1$, if $i = x$; otherwise $J_i(x) = 0$.

What really makes non-Cartesian logics interesting from philosophic point of view is that these logics do not allow direct use of algorithm, formulated by Caleiro and others in [1], for construction of bivalent semantics. This algorithm may be seen as an attempt of constructive realization of Suszko's Thesis, but due to pure existence of non-Cartesian logics one can immediately conclude that this algorithm is far from being universal. This, in its turn, may be viewed as a support to the hypothesis that Suszko's reduction cannot be universally constructive in principle.

Sm_4 , however, allows a standard Hilbert-style axiomatization, which consists of axioms and rules of the system E (of entailment) plus a single axiom and two deductive rules:

A+: $A \& \sim A \rightarrow B$;

R+1: $\sim A / A \rightarrow B$;

R+2: $B / A \rightarrow B$.

Such an axiomatization is semantically adequate for Sm_4 , this can be proven using standard methods.

References

1. Caleiro C., Carnielli W., Coniglio M. E., Markos J. “Two’s company: the Humbug of Many Logical Values” [in:] *Logica Universalis*, Birkhauser Verlag, 2005. Pp. 169 -189.
2. Karpenko A. “Development of Many-Valued Logic”, Moscow: URSS, 2010 (in Russian).
3. Prelovskiy N. “On One non-Cartesian Extension of System E” [in:] *Logical Investigations*, V. 17, Moscow, 2011. Pp. 251-255 (in Russian).
4. Shramko Y., Zaytsev D. “Logical Consequence and Designated Values” [in:] *Logical Investigations*, V. 11, Moscow, 2004. Pp. 119-125 (in Russian).
5. Shramko Y., Wansing H. “Logic of Computer Networks” [in:] *Logical Investigations*, V. 12, Moscow, 2005. Pp. 119-145(in Russian).

Note

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Pragmatico-Linguistic and Semiotic Tools in Analysis of Electronic Conversation

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Abstract:

In the paper I show that the tools of pragmatical analysis of face-to-face conversation can be easily used also and developed in researches concerning e-conversation.

CMC-studies researchers do not pay their attention on methods of pragmatics (here theory of conversation) probably because the Internet in its communicative aspect is treated as textual medium or hypertextual one, and because communication via the Internet is often seen as 'impersonal' (Wood, Smith [2005]). Users of the electronic communication channel usually do not see each other, hence there is no non-verbal communication between them – they send text messages constructed and displayed with the use of given software. Pragmatico-linguistic analyses have been developed in an area of philosophy of language (J. Austin, J. Searle, H.P. Grice) and psycholinguistics (H.H. Clarke) and those scientific disciplines did not (and obviously could not) deal with online communication/conversation, and they were out of the scope of interest of CMC-studies scientists.

Pragmatists analyse face-to-face conversation and in their concern there are 1) different contexts of such conversation (namely: linguistic, situational, interpersonal, cultural and cognitive ones), 2) processes of conversational negotiation of meaning, 3) presuppositions (hidden assumptions of conversation) and 4) the structure of conversation. The rich and complicated tools of pragmatics at first glance seem to be unuseful in any analyses of e-conversation in which interlocutors do not see each other and quite often do not know each other as well, and moreover CMC-studies researchers point to asynchronicity of electronic communication or conversation, that means existing of time periods between some sent messages. In other words in e-conversation a synchronical exchanging of messages is rare and there are some technological constraints that do not allow the Internet users to send their messages in e-conversation at the same time (whereas talking people can utter their sentences simultaneously): texts that are sent are displayed on screens in chronological way, one after one.

However when we take into account that 1) e-conversation is performed to reach the same goals as our usual conversations in real world, 2) in its textual layer and its informal shape e-conversation is similar to 'talking', 3) interlocutors themselves 'record live' their exchange of written utterances (e-utterances), we can try to reconstruct the structure of online conversation. Every interpersonal communication should have some elements that can be discovered no matter which medium is used by interlocutors in their communication process. Those elements are reconstructed by linguistic pragmatists.

And what about the impersonal feature of e-conversation? We can generally assume that there are some elements of nonverbal communication which belong to the set of meta-textual signs (emotional icons, gifs, pictures etc.). Those signs function more less as discourse markers but also

as short comments or remarks sent to receivers to 1) simplify processes of interpreting messages and meaning-negotiations, 2) weaken the communicational 'rawness' ('impersonality') of pure text. In netiquette when a user writes his messages using capslock it is common to interpret it as his 'shouting' online. When he 'floods' (sending lots of texts just to cover the screen when other users chat) it is treated as wordiness or even 'trolling' (disturbing/interrupting of communication). Interlocutors can change colours of signs etc. We should remember however that those iconic means are very poor in comparison with cues of nonverbal communication. It would be better to understand emoticons etc. only as meta-signs that are used by interlocutors on meta-conversational level and which express a sender's attitude to a message rather than to a receiver (in contrast to intentions that matter in any natural conversation and can be expressed with someone's mimic or gestures).

Since the 'non-verbal' layer of e-conversation is simply iconic, then it can be analysed by the means of semiotics. The sign :) does not make any communicative or interpretative problems for the Internet user, it is also one of the most popular (in its emotive function) icons which help a receiver of a message interpret the message accurately. We should bear in mind that although the signs like emoticons are not linguistic expressions at all, they are treated by interlocutors as necessary elements of e-conversation. In that way those signs can be apprehended as some kind of analogs of our eye or face expressions. But we should not see any analogies or similarities where they are absent. The whole meta-conversational layer of emoticons etc. is a highly conventional and arbitrary code, whereas in our ordinary talks the nonverbal layer is often quite natural which we do not have to learn before we start communicating face-to-face with someone else.

In pragmatics we distinguish following elements of the structure of conversation: 1) adjacency pair of utterances/sentences, 2) pre-sentences, 3) discourse markers and 4) grounding. Any conversation is possible when two people exchange each other one sentence at least – hence a pair of sentences is the smallest unit of conversation. Pre-sentences are to initiate a conversation or one of its topic, they also may establish a goal of the conversation (pre-requests, pre-invitation, pre-announcements). If a conversation is to develop fluently, dynamically, interlocutors during turn-taking use discourse markers to fasten or slow down a tempo of the conversation. The most important is grounding however, since any fruitful or effective conversation requires from its participants to make conversational moves on their common ground of cognition, knowledge, experience, beliefs cultural context etc.

The semantical and contextual spheres of conversation are not the end of story. Pragmatists say that every conversation has a hidden layer which is communicated but not expressed verbally. Even a speaker or a listener both make assumptions intentionally connected with uttered/heard sentences by them, thus every conversation is accompanied by some conversational inferences (performed by interlocutors) which deal with what is communicated 'between the lines', what is communicated 'at the back' of uttered expressions, what is unsaid but somehow communicated. The layer consists of presuppositions implied by the sentences exchanged during the conversation.

The presuppositions (accordingly to Yule [1996]) are existential, factive, non-factive, counter-factive, structural and lexical. Since our conversations usually refer to real people, things, events etc. we tacitly assume that referents/designates/states of affairs etc. of the sentence uttered or heard by us exist actually. Thus when someone says: *Dorothy lives in an exclusive block of flats in Krynica*, we tacitly assume (and these are the existential presuppositions) that 1) the Dorothy is a real person, 2) the block of flats actually exists, and 3) Krynica as a Polish town, as well. The factive presupposition here is that Dorothy really lives in that block. When we hear someone speaking: *I didn't know that Kate had changed her job*, we infer from the sentence that Kate changed her job (the factive presupposition). When someone says: *I dreamed about being a wealthy man*, we assume that the speaking person is not wealthy (the non-factive presupposition). When we hear: *If the Smiths had loved each other, they would not have divorced last year*, we assume that the Smiths did not love each other, especially last year (the counter-factive presupposition). When we ask: *Why hasn't Helen come to the party?*, our interlocutor assumes that Helen has not come to the party, because the structure of the uttered question itself implies such (structural) presupposition.

When we say: *Paul quitted smoking*, the expressions used in the sentence imply (the lexical presupposition) that Paul used to smoke some time ago.

Usually there are a couple of presuppositions connected or correlated with every uttered sentence, but there can also be such conversational situations wherein a sentence implies different presuppositions of the same type at the same time. It can happen for example when we use a verb which has different meanings in different co-texts and contexts – when someone says: *Tom has found the CD at last*, we can simultaneously draw conclusions (here the conversational presuppositions or entailments) that 1) Tom had been looking for the CD (in music shops, in the Web etc.) before, 2) Tom had lost the CD (during his tidying or removal to another house) until he happily found it. In the case of simultaneous implying different presuppositions by one sentence the way to find the right interpretation of the sentence is to get to know the contexts (especially linguistic and situational ones) which can help us eliminate inaccurate presuppositions.

The tools of pragmatical analysis of face-to-face conversation can be easily used and developed in researches concerning e-conversation, moreover, in CMC-studies we can find a few advantages that do not exist in situations of ordinary talks: 1) e-conversations are recorded by their participants themselves during communication (hence it is easy to use them as an empirical (and electronically archived) material for further analyses), 2) a researcher can easily observe interlocutors (as an anonymous chat user who does not participate in a given e-conversation) and they do not mind their being observed (in contrast to natural situations in which people do not want to be observed or when observed they talk artificially or stop freely talking at all), hence 3) there are no ethical constraints to such participant observation.

References:

1. Carol, John M., *HCI Models, Theories, and Frameworks: Toward a Multidisciplinary Science*, Morgan Kaufmann, San Francisco 2003.
2. *Conversation and Cognition*, ed. by Hedwig te Molter and Jonathan Potter, Cambridge University Press, Cambridge 2005.
3. Duranti, Alessandro, *Linguistic Anthropology*, Cambridge University Press, Cambridge 1997.
4. Grice, Herbert Paul, *Studies in the Way of Words*, Harvard University Press, Cambridge 1989.
5. „Utterer's Meaning and Intention”, [in:] *The Philosophical Review* Vol. 78, No. 2 (1969) 147-177.
6. *Research Methods for Human-Computer Interaction*, ed. by Paul Cairns and Anna L. Cox, Cambridge University Press, Cambridge 2008.
7. Schwartz, Norbert, *Cognition and Communication. Judgmental Biases, Research Methods and the Logic of Conversation*, Lawrence Erlbaum Associates, Mahwah 1996.
8. Searle, John R., *Expression and Meaning. Studies in the Theory of Speech Acts*, Cambridge University Press, 1979.
9. Stalnaker, Robert C., *Common Ground*, [in:] *Linguistics and Philosophy* 25 (2002), pp. 701-721.
10. Wood, Andrew F. and Smith, Matthew J., *Online Communication. Linking Technology, Identity and Culture*, Lawrence Erlbaum Associates, Mahwah 2005.
11. Yule, George, *Pragmatics*, Oxford University Press, Oxford 1996.

The Dial of the Circular Complementarity of the Designated and Antidesignated Pairs

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Abstract:

The priority is given to MACROLEVEL, and MICROLEVEL only in the absence of macrodominant. Variable elements A, u form a logic wave, and invariable elements V, n – logic atom.

Keywords:

multiple-valued logic, microlevel and macrolevel, logic atom and logic wave, circular complementarity.

He who possesses Geometry bears it away, and acquires a new vigor.

B. Pascal. Of the Geometrical Spirit

The epistemological lesson of atomic physics has naturally, just as have earlier advances in physical science, given rise to renewed consideration of the use of our means of communication for objective description.

N. Bohr. Atomic Physics And Human Knowledge

“Let None But Geometers Enter Here” – that was a motto of Platonic Academy. The founder of the higher geometry chair in Sorbonne M. Chasles said that the geometry “is considered as a basis of mathematical sciences, and the best thinkers of all times considered it, as an excellent exercise in the logics, extremely suitable for development of great minds” [17, p. 515]. Lobachevsky's works have focused scientists' attention on the problem of the relationship between various geometrical constructions. “When analyzing formation of a principle of compliance in the history of geometry, usually the value of ideas of N.I. Lobachevsky for identification of relationship between Euclidean and not Euclidean geometry is appreciated” [12, p. 234]. The geometry is the most ancient mathematical discipline, and the higher geometry (which is usually called projective geometry) has shown how to unite both classical and non-classical theories [8, p. 242]. Empirical nature of geometry and logic was noted many times. S. Kleene emphasised similarity of geometry and logic in his *Mathematical Logic*. It is a good example of convergence of various scientific disciplines. Some logicians can tell: “I see no logic!” As in case of geometry the synthesis of classical and non-classical logics requires **the highest logic** which is generated by projective interpretation of a Boolean polysemy [4, pp. 14 – 34]. It should be noted that contemporaries did not quite understand G. Boole's logic which was considered in details by N.I.Styazhkin as non-classical, multi-valued logic [14, pp. 329, 335].

The principle of compliance establishes relation between classical and non-classical approaches. Asymptotic coincidence of classical and non-classical theories is the first manifestation of **the correspondance principle**. When arranging true-false pairs we consider them equivalent. According to **the correspondance principle** the designated and antidesignated pairs should be equivalent – *fifty-fifty*. At the same time the principle of compliance was one of sources of **the complementarity principle**.

First of all it deals with a complementarity of description levels carried out through essentially different experimental concepts. The concepts of microlevel and macrolevel as a methodological principle have been offered firstly in my article for magazine «Philosophy matters» in 1970 that was appreciated by corresponding member A.A. Lyapunov and was prepared for publication, but then its galley proof was withdrawn (because of quotations from A.A.Zinovyev's work). The complementarity of levels generates a **fractality** (self-similarity) of the main table structure. **BLOCKS** of macrolevel *are subdivided* into microlevel cells. All two-letter words are formed by application of the “letters” to themselves, providing a quantum leap to the highest level [6].

Using a traditional arithmetization of logic where 1 designates truth and 0 designates false, we have **DOMINANTS** which are *the absolute maximum* A = 11 which is designated value and the *absolute minimum* V = 00 which is antidesignated value. **NONDOMINANTS** are the designated u = 10 and the antidesignated n = 01. These four values form pairs (x, y) where the prefix x describes MICROLEVEL, the root y describes MACROLEVEL. When deciding on the designation of the pair (MICROLEVEL, MACROLEVEL) the priority is given to MACROLEVEL, and MICROLEVEL is chosen only in the absence of a macrodominant.

MICROLEVEL:

n*	A*
V*	u*

-
↓

MACROLEVEL

*n	*A	→	nn	An	nA	AA
*V	*u		Vn	un	VA	uA
			nV	AV	nu	Au
			VV	uV	Vu	uu

Let's accept the complementarity concept. A. Petersen considers its historical and philosophical roots in the problem of stability and changeability [20, p. 62]. Divisible, changeable elements form a logic wave, and indivisible ones form stable elements – logic atom which has the fourfold duplication that provides the higher safety when transferring genetic information.

Strong pair implies macrolevel priority and weak pair provides microlevel priority. Diagonal oppositions anti-commutate in designation that generates a complementarity.

The complementary pair of vowels A, u creates a *wave*, and the pair of consonants V, n forms **ATOM**. Adjacent pairs commute in the identity on designation: **Au** = **uA**. Diagonal oppositions of the designated and antidesignated true-false pairs (wave-particle and particle-wave) anti-commutate in identity on designation: **AV** = – **VA**. AV has the beginning A specifying the 9th evenings, and **VA** – the beginning V for the 9th mornings.

nn	<u>An</u>	<u>nA</u>	<u>AA</u>	=	01 01	11 01	01 11	11 11	night	3/	6 morning	9	12	day
Vn	<u>un</u>	<u>VA</u>	<u>uA</u>		00 01	10 01	00 11	10 11		n ₃	n ₆	A ₉	A ₁₂	
nV	AV	nu	<u>Au</u>		01 00	11 00	01 10	11 10		V ₁₂	V ₉	u ₆	u ₃	
VV	uV	Vu	<u>uu</u>		00 00	10 00	00 10	10 10		12	) evening	6	/ 3	

Diagonal oppositions – midday and midnight – are in antiphase as antipodes on the offered dial of the designated and antidesignated pairs. These are complementary a complementarity.

The anti-commutativity does not hide non-identity of the paradigm and anti-paradigm, but prepares qualitative transfer. Here **the complementarity principle** is fundamental, and N.Bohr considered nonclassical noncommunicativity as its most essential manifestation. “The noncommutativity formula turned from striking absurd into the unforeseen guarantee of the fruitfulness of the discovered way” [7, p. 259].

Von Weizsäcker was the first who paid attention that complementary descriptions are in the circular relation. He associates the complementarity concept with the general gnoseological model of “the circle of knowledge”. [1, pp. 159, 193]. It is necessary to join his opinion that this concept is in essence logical. The dial describes a circular order of the genetic code diagrams and socionics quadras well [5, pp. 167-174]. Diagonal oppositions are complementary. “They teetered on scales: the morning raised one, the evening raised another” [9, p. 349]. There are good grounds for Russian saying: “The morning is wiser than the evening”. The complementary pairs appears on the macrolevel not together. Thus the overlaps from right to left and from left to right form a hysteresis loop. This so-called “inertia” was noticed long ago by experts in psychophysics.

The flying arrow achieves the target in multilevel logical model. Though it is motionless at every moment of time, but movement process as result of merging discrete frames is not a paradox (as Zeno stated) but a logical vector. It is particle-wave [2, p. 34]. The specified triad forms CINEMA = (freeze frames, movement, phase). Leibniz’s actual infinitesimalls were proved within non-standard mathematical analysis. That is the atomism of knowledge. A logical codon (MICROLEVEL, MACROLEVEL, *phase*) includes a fluxion *z* which describes a wave phase.

Let's count up the total number of distinguishable (by amino acids) logic codons. There are 8 indivisible codons + 8 x 2 doubled codons = 24 codons, distinguishable by amino acids, minus 3 repeated codons result in 21 = 20 amino acids + the STOP command.

The consideration of non-classical logic in terms of the classical logic allows to show clearly a role **of the designated values** for classification of the trigrams by means of the digram matrix of the genetic code. At first a logic matrix is constructed of large blocks, and then some cells are arranged and filled according to the principle of similarity [3, pp. 53-59]. It was possible to construct fractal cards due to the fact that positional recording requires just 4 letters at the highest levels.

The principle of a fractality must succeed within our Solar system! The Earth’s rotation around its own axis and its rotation around the Sun can serve as a bright example of a temporary fractality: *day periods* (morning, day, evening, night) are similar to *seasons* (spring, summer, autumn, winter). Plato prophetically stated: “The reason for God to invent and give us sight to the end that we might behold the courses of intelligence in the heaven, and apply them to the courses of our own intelligence which are akin to them...” [11, p. 450]. As a result Ptolemaeus’s epicycles emerged for heavenly bodies and al Arabi’s concentric circles for our thinking. Its sectors (*Speaking, Loving, Knowing and Dominating*) does not casually seen similar to the modern classification in the socionics [9, p. 69].

Aristotle’s LOGICAL SQUARE quartered Natural Universal but “presented” to it a prison cell with a square outlook. Unlike the European astrological charts, of the rectangular shape, the Arabian charts **traditionally had the round shape**. Later the circular order prevailed in Europe as well. The striking examples are the round seal inherited from alchemists, and R. Lully’s logical machine. Now the matrix of complementarity provided the circular arrangement (*n “spring” – A “summer” – u “autumn” – V “winter”*) in MATRIX OF COMPLEMENTARITY unlike the LOGIC SQUARE:

n spring A SUMMER
V WINTER u autumn

n spring A SUMMER
u autumn V WINTER

Aristotle applied the logic square in syllogistics where partial affirmative and partial negative judgments were nearby in one column, and general affirmative and general negative judgments are nearby in other column. Figuratively in that case summer and winter are nearby. A .Koyre believed that “the theoretical thought and human life are separated by a chasm” [19, p. 43]. He underlined this thesis, stating that the world of the science is leaving and separates from the living world. In the **living logic** the Matrix of Complementarity does not allow that gap. *The GENETIC APPROACH IN LOGIC* allows to define the features matching the **plan of the Nature**.

When creating the universal language the similarity of the trigrams and triplets of the genetic code can help. That was discovered firstly by the Nobel prize winner in molecular genetics F. Jacob. The genetic code is an information code. In philosophy of a science there is no other way to be loyal to the Nature, except to be loyal to genetics. Analytism is based on the positioning principle that should be applied for humanities because it offers there not less advantages than for arithmetics. It realizes Leibniz's dream – to make mathematics universal language in fact. “Universal” literally means “turning into one” (from Latin *Unus* = one and *Versus* – a participle from *Vertere* = to rotate). It is “the ability of one to turn different sides, ... a plenty in one subject” [18, p. 643]. In fact there is turning when turning!

“Since Plato the western thought and the theory of knowledge have concentrated on concepts True/False. However it is high time to shift to Stable/Unstable, and in a social Epistemology – to more serious problem of Dupe/Egghead...The most estimable task is to devote oneself to what has been neglected for very long time – to creation of the charts that could define limits of our current knowledge and our current methods” [15, pp. 206, 117-118]. N. Taleb emphasizes: “I used the concept ‘bulge’ – disproportionate nonlinear reaction to change of basic data when all tools to measure the accuracy level may be thrown out safely” [15, p. 102]. He applied the term “Black swan” for rare but shocking crises in the. This block of *the bulgy dissidents* allows to describe **variability of *A** as a cluster equivalent to a cluster of **stability *V** – *caved in to the authority* concordants.

The problem “how to shift from one style of thinking to another” was set by L. Fleck emphasizing the importance of the social and cultural aspect in philosophy of a science [16, p. 55]. He influenced greatly on the concept of T. Kuhn’s abrupt transitions to whose emphasis only on the ‘Normal’ science *without any irony* [13, p. 144] did not allow him to describe the structure of the scientific revolutions. However even L. Fleck repeatedly emphasized the impossibility of formal and logical interpretation of the cognitive process [16, pp. 37, 57, 61, 75]. The concept of a social clustering can help. However its author Academician V.L. Makarov specifies *the only cluster* – rigidity (“skeleton”), opposing it to softness (“muscles”) [10, p.11]. However only after consideration of **two dominant clusters** *V and *A, it is possible to plan solutions of the PROBLEM of CHANGES.

The logical **positioning** can help essentially to humanists in creation of universal language. Its letters can be small and big (dominants), concave and bulgy (“not caved in”) concordant and public (dissidents). That’s why A. Makarevich sang: “Don’t cave in to the changing world, get the world to cave in to you”. This motto practically realizes sociocultural aspect in the philosophy of science.

The offered genetic method to solve the problem of transition from one style of thinking to another is the *message* which provides the clue to cognitive process that will help fill the gap between natural intelligence (NI) and artificial intelligence (AI).

Reference

1. Alekseev I.S. *Koncepciya dopolnitel'nosti*. Moscow, 1978. P. 159, 193.
2. Bahtiyarov K.I. *Logika s tochki zreniya informatiki*. Moscow, 2002. P.34.
3. Bahtiyarov K.I. "Kvadrat Protivopolojnosti i Matrica Komplementarnosti" [in:] *Polignozis*. 2009, № 4. P.53-59.
4. Bahtiyarov K.I. "Kak mojet rassujdat' komp'yuter v duhe bulevoi mnogoznachnosti" [in:] *Logicheskie issledovaniya*. Vyp. 17. Moscow, 2011. P. 14-34.
5. Bahtiyarov K.I. "Geneticheskii metod dlya stilei myshleniya" [in:] *Metafizika*. Moscow, 2012. P.167-174
6. Bahtiyarov K.I. *Logika i psihogenetika s tochki zreniya informatiki*. Moscow, 2013.
7. Danin D.S. *Nels Bohr*. Moscow, 1978. P.259, 349.
8. Efimov N.V. *Vysshaya geometriya*. Moscow, 1971. P.242
9. Ibn al-Arabi. "Izobrajenie okrujnostei, ohvatyvayuschih podobie cheloveka Tvorcu i sotvorennomu miru" [in:] Ibn al-Arabi. *Mekkanskije otkroveniia*. Sanct-Petersburg, 1999. P.69.
10. Makarov V.L. *Social'nyi klasterizm*. Moscow, 2010. P.11.
11. Platon. *Timei* [in:] *Soch.*, t.3., M., 1994. P.450.
12. *Princip sootvetstviya*. Moscow, 1979. P.234..
13. Prujinin B.I. *Ratioservens. Kontury kul'turno-istoricheskoi epistemologii*. Moscow, 2009. P.144.
14. Styajkin N.I. *Formirovanie matematicheskoi logiki*. Moscow, 1967, P. 329, 335.
15. Taleb N. *O sekretah ustoichivosti*. Moscow, 2012. P. 102, 110, 117-118, 206.
16. Flek L. *Vozniknovenie i razvitie nauchnogo fakta. Vvedenie v teoriyu stilya myshleniya i myslitel'nogo kollektiva*. Moscow, 1999. P. 37, 55, 57, 61, 75.
17. Shal' M. *Rukovodstvo vysshei geometrii*. Moscow, 1910. P.515.
18. Epshtein M. *Znak probela*. Moscow, 2004. P.643.
19. Koyré A. *Etudes new tonniennes*. Paris, 1968. P. 43.
20. Petersen A. *Quantum physics and the philosophical tradition*. Cambridge, 1968. P. 62.

Is the Polish Logic One of the Best Traditions Still?



Roman Murawski is Professor at Faculty of Mathematics and Computer Science of Adam Mickiewicz University, Poznań, Chairman of the Department of Mathematical Logic, former President of Polish Association for Logic and Philosophy of Science.

Andrew Schumann: The Polish logical tradition is one of the best. How can you explain the fact that Polish philosophers and mathematicians have been a long way in logic and analytic philosophy? Which Polish scientific centers are still heavyweight in this subject?

Roman Murawski: One should look for roots in the interwar period. Polish logic and analytic philosophy at that time is an amazing phenomenon. The school has been founded by Kazimierz Twardowski and is called Lvov-Warsaw school of philosophy. A part of it was also Warsaw school of logic. There is a fundamental monograph (published by Kluwer) describing this school and its achievements – I mean Jan Woleński's *Logic and Philosophy in the Lvov-Warsaw School*. The standards of a scientific work in logic and analytic philosophy developed then certainly helped to reach the level you are talking about. According to Twardowski and his students, one should clearly and sharply distinguish world-views and the scientific philosophical work. This idea was particularly stressed by Łukasiewicz, the main architect of the Warsaw school of logic. He regarded various philosophical problems pertaining formal sciences as belonging to world-views of mathematicians and logicians but the work consisting in constructing logical and mathematical systems together with metalogical and metamathematical investigations constituted for him the subject of logic and mathematics as special sciences. Hence philosophical views cannot be a stance for measuring the correctness of formal results. Yet philosophy may serve as a source of logical constructions. One should disregard philosophical controversies (and treat them as a „private” matter) and investigate (controversial) axioms as purely mathematical constructions using any fruitful methods.

An interesting phenomenon was also the close collaboration of philosophers, logicians and mathematicians (especially in Warsaw) which resulted in important achievements.

Andrew Schumann: Which contributions of Polish logicians to decidability theory and recursion theory could you notify as the most important?

Roman Murawski: One should start by mentioning the method of quantifier elimination studied by Tarski and his students. This method had various applications to the decidability problems. Using this method Tarski proved the decidability of the theory of Boolean algebras, of the theory of dense linear order and of the theory of discrete order. He applied it also to the study of geometry and to the field theory showing the decidability of the first order theory of reals. He proved also the decidability of the theory of real-closed and algebraically closed fields. Among the decidability

results obtained by quantifier elimination method by Tarski's students the most famous is Moses Presburger's result on the decidability of the arithmetic of addition. Tarski, together with his student Andrzej Mostowski showed the decidability of the theory of well ordering.

Polish logicians considered also and showed the undecidability of various theories. One should again mention here Tarski and his work on general methods of establishing the (essential) undecidability of first order theories. Using those methods the (essential) undecidability of various theories has been shown. One should mention here also the finitely axiomatizable arithmetic Q developed by Tarski, Mostowski and Robinson which appeared to be very useful in decidability studies.

One should mention also works by Józef Pepis on reducibility. Unfortunately Pepis was killed by Gestapo, probably in August 1941.

What concerns the contribution of Polish logicians to the recursion theory one must mention first of all the paper by Andrzej Grzegorzczuk where a hierarchy of primitive recursive functions has been introduced and studied. This hierarchy is called today Grzegorzczuk's hierarchy. It has been carefully studied by various logicians, it has been extended and generalized. One found various applications of it also outside logic, in particular in theoretical computer science and the complexity theory.

As next contribution of Polish logicians to the recursion theory one should mention the classification of non-recursive relations constructed independently by S.C. Kleene and Andrzej Mostowski and called today Kleene-Mostowski hierarchy. Let us mention also Grzegorzczuk's studies of computable functionals of higher types as well as Banach-Mazur's and Grzegorzczuk's studies on constructive mathematics and Mostowski's and Grzegorzczuk's studies on the complexity of models of theories.

Andrew Schumann: What can you state about the development of Hilbert's program? Is it failed as the majority think?

Roman Murawski: Gödel's incompleteness theorems indicated certain difficulties in carrying out the validation and justification of classical mathematics on finitistic grounds postulated by Hilbert. They struck Hilbert's program but they did not reject it. The natural consequence of it was the idea of extending the admissible methods and allowing general constructive methods instead of finitistic ones. It seems that Paul Bernays was among the first to recognize this need. The very concept of constructive methods is in fact not quite clear. Nevertheless the idea has been accepted and became a new paradigm leading to the so called generalized Hilbert's program. Investigations were carried out in this direction and several interesting results have been obtained. One should mention here studies that followed Gentzen's idea of using transfinite induction on a certain recursive ordering (Schütte, Takeuti), the program of predicative reductionism (Feferman) or the idea of using primitive recursive functionals of higher types (Gödel). One should add that all those attempts are in fact different from the original Hilbert's program. Hilbert postulated the justification and validation of classical mathematics by a reduction to finitistic mathematics. This had an important philosophical meaning: finitistic objects and reasoning have a clear physical meaning and are indispensable in all scientific thought. None of the proposed generalizations can be viewed as finitistic and they do not have a similar philosophical and methodological meaning. Nevertheless the generalized Hilbert's program is an interesting contribution and is compatible with Hilbert's reductionist philosophy.

Another consequence of Gödel's incompleteness theorems is the so called relativized Hilbert's program. If the entire classical mathematics cannot be reduced and justified by finitistic mathematics then one can ask for which part of it is that possible? In another words: what part of classical mathematics can be developed in formal systems that are conservative over finitistic mathematics with respect to real sentences. One of contributions to this program is the reverse mathematics initiated by Harvey Friedman. Results obtained within this research program lead to

the conclusion that a large and significant part of classical mathematics is finitistically reducible. This means in fact that Hilbert's program can be partially realized.

Andrew Schumann: What are mechanized deduction systems in fact? Why are they being constructed? What can be provided by their implementations and where?

Roman Murawski: In 1936 Alan Turing and Alonzo Church proved two theorems which seemed to have destroyed all hopes of establishing a method of mechanizing reasonings. Turing reduced the decidability problem for theories to the halting problem for abstract machines modelling the computability processes (and named after him) and proved that the latter is undecidable. Church – solving Hilbert's original problem – proved the undecidability of the full predicate logic and of various subclasses of it.

On the other hand results of Skolem and Herbrand showed that if a theorem is true then this fact can be proved in a finite number of steps – but this is not the case if the theorem is not true (in this situation either one can prove in some cases the falsity of the given statement or the verification procedure does not halt). This semidecidability of the predicate logic was the source of hope and the basis of further searches for the mechanized deduction systems. Those studies were heavily stimulated by the appearance of computers in early fifties. There appeared the idea of applying them to the automatization of logic by using the mechanization procedures developed earlier. The appearance of computers stimulated also the search for new, more effective procedures.

The idea is here to use a computer to prove non-numerical results, i.e., to determine their truth or falsity. One can demand and expect either a simple statement „proved” or a human readable proof. We can distinguish also two modes of operation: fully automated proof search or man-machine interaction proof search.

Note that the studies of mechanized deduction systems were motivated by two different philosophies. The first one – call it logic approach – can be characterized by using of a dominant logical system that is delineated and in fact static over the development stage of the theorem proving system. The second philosophical viewpoint is called the human simulation approach. It is generally the antithesis of the first one. Here one attempts to simulate human techniques of solving problems. Of course the logic and human simulation approaches are not always clearly delineated. Various mechanized deduction systems have been developed. Let us mention here systems of Davis, Newell-Shaw-Simon, Gilmore, Gelernter et al., Hao Wang and Davis-Putnam. Very important role is played in those research also by the resolution and unification algorithms of Prawitz and Robinson. They turned out to be crucial for the further development of the researches towards mechanization and automatization of reasonings.

What does one expect from mechanized deduction systems and from an automated theorem prover? First of all certain unification of reasonings and their automatization are obtained. If one has such a system or prover one can shift the burden of proof finding from a mathematician and a logician to the computer. In this way one is also assured that faulty proofs would never occur. There is a question whether such automated theorem provers are clever than people? Of course they can proceed quicker than a human being. But they can also discover new mathematical results. In fact some open questions have been answered in this way within finitely axiomatizable theories. On the other hand there are some limitations implied by theorems on the complexity of decision procedures.

Andrew Schumann: What is reverse mathematics and which philosophical meaning does it have?

Roman Murawski: Reverse mathematics is a research program formulated by Harvey Friedman in 1974. Its aim is to study the role of set existence axioms, i.e., comprehension axioms in ordinary mathematics. The main problem can be formulated as follows: Given a specific theorem T of ordinary mathematics (e.g., of analysis, of algebra, of functional analysis, of differential equations, etc.) ask which set existence axioms are necessary in order to prove T? The procedure used in the

reverse mathematics (and explaining its name) is to show that the considered theorem T is in fact equivalent to the existence axioms used in the proof of T and the main and usually most difficult part of the proof is to show that T implies the axiom (hence the procedure is here in a certain sense a reverse of the usual procedure used in mathematics where one proves that a given axiom implies a theorem). Some specific systems have been considered here (they are in fact subsystems of the second order arithmetic with various forms of the comprehension axiom) and their role and meaning with respect to various theorems from „hard” mathematics have been investigated leading to many very interesting results. Unfortunately they are rather very technical and complicated and it is impossible to describe them here in detail. One of the consequences of those results was the corollary indicated already above that Hilbert’s program can be partially realized.

Andrew Schumann: Whether there can be a logical symbolism for anything? What philosophical background does symbolism have as a whole?

Roman Murawski: There are three kinds of motivation inspiring the development of symbolism in logic: (1) the attempt to create an ideal artificial language as a substitute for an imprecise colloquial language, (2) a tendency to reduce logic to the study of properties of language or, in extreme cases, to the theory of signs, (3) a nominalistic tendency according to which abstract terms do not denote objects but are only empty signs. One or more of those tendencies can be seen in all logicians trying to develop a symbolism in logic. For example Aristotle exemplifies the first tendency, in the Stoics one sees clearly linguistic tendencies and in mediaeval logic one sees some semiotic tendencies (Abelard, the nominalists, Ockham).

There is a problem of relations between symbols and reality. It has been solved in various ways by logicians. One should mention also the tendency to overestimate the role and significance of symbolism. In this context one can mention the great Polish philosopher, the founder of Lvov-Warsaw School of Philosophy, Kazimierz Twardowski and his paper “Symbolomania i pragmatofobia” [Symbolic mania and pragmatic phobia] where he emphasized that symbols represent always objects but cannot replace them. A symbol is only a tool. If one forgets these two things we have the attitude Twardowski called symbolic mania. It can be characterized by a faith in the infallibility of a symbolism, in an autonomy of operations on symbols and by a condemnation of opinions which are independent of any symbolism. This attitude is connected with another called by Twardowski pragmatic phobia and consisting of bias against objects denoted by symbols.

Andrew Schumann: What is mathematical or logical truth? Does a mathematical or logical reality exist outside of the life-world?

Roman Murawski: Well, the usual and in fact the unique method of establishing truth in mathematics and logic is to construct a proof. The very concept of a proof is not quite clear and rather vague. In mathematical research practice the role of a proof is to convince other mathematicians that a given statement holds. Logicians tried to make it more precise by introducing the concept of a formal (or formalized) proof. But is it an adequate counterexample of proofs from mathematical practice?

Gödel’s first incompleteness theorem shows that one should distinguish between provability and truth (in a given model). In fact what can be proved is true but not always vice versa. Hence there are sentences that are true (in a given model) but that are simultaneously undecidable, i.e., neither they nor their negations can be proved in a considered theory. What means here “true” (in a given model) was explained by Tarski in his famous definition from 1933. His definition is connected with the classical definition given by Aristotle and called the classical definition of truth or the correspondence definition. It says that a sentence is true if and only if it adequately describes the state of affairs in the reality. In the case of a mathematical sentence one should speak about the mathematical reality. But what it is? Here we come to the second part of the question. This is one of the most fundamental problems of the philosophy of mathematics and logic. Several answers have

been given here. One can classify them into three main groups. The first says that mathematical objects exist in an objective way and are independent of time, space and human mind. They are given to a mathematician and logician whose aim is to discover them and to describe their properties and mutual relations. One calls this doctrine Platonism. Another one says that mathematical objects exist in fact in human mind and are mental construction of mathematicians. This idea is called conceptualism. The third one called nominalism claims that there are in fact no abstract and ideal mathematical objects – there exist only physical items and in mathematics (and logic) we have to do only with expressions that should be treated as physical objects. All those doctrines have their adherents. One should add however that normal mathematicians behave in their research practice usually as platonists being convinced that the mathematical reality is given to them and that they do not have an unlimited freedom in dealing with mathematical objects they are studying. Note also that a philosophical declaration with respect to the problem of existence in mathematics can imply a limitation of admitted methods and considered problems (as it is a case by intuitionism) or one can treat philosophical sympathies as a private matter and develop mathematics or logic using any correct methods (as it was by Polish logicians and mathematicians in the 1920s and 1930s).

Charting the Sea of the *a fortiori*
Review of the book

About: Ury, Yisrael, *Charting the Sea of Talmud. A Visual Method for Understanding the Talmud*, Mosaica Press, Jerusalem, 2011.

Stefan Goltzberg

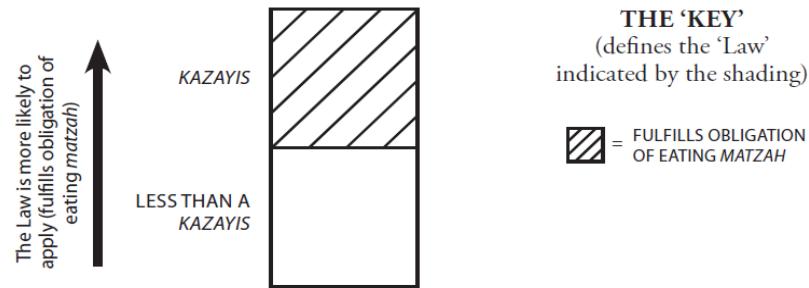
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The Talmud is a sea, a sea into which one can dive or be drowned in or simply observe carefully from the shore.¹ And this is not any sea, but one of the most opaque seas. Indeed, one definitely should be accompanied in the sea of Talmud. This is the purpose of this excellent book. However, while methods of explaining the Talmud are usually textual, Yisrael Ury's method is different; he creates Diagrams² to explain difficult topics.

Why change the medium, though? Isn't explaining texts through images, making things more complicated? In fact, it makes things easier: "Vision is the sense that conveys information most clearly" [17]. The objective of this book is "to introduce the Talmud Diagram, a novel visual tool, which simplifies the tracking of connected facts in the Talmud" [18]. It is not just a way of visually illustrating what happened at the time of the Talmud or depicting objects that are talked about in Talmudic texts. Indeed, there are already many books that visually represent the situations described in the Talmud: Ury's goal is "to visually represent the **logic** of the Talmud since this is often the hardest part of the *sugya*³ [Talmudic discussion] to understand" (20).

How does it work exactly? The main tool is the so-called "Talmud Diagram." "The Talmud Diagram is a unique type of table where placement has meaning and helps explain a *sugya*" (20). Before explaining the typical Talmudic Diagram, let us take an example of a simple Diagram: in order to fulfill the obligation of eating *matzah* on Pesach, it is necessary to eat a volume of *matzah* the size of a *kazayis* (an olive). This implies that if you ate less than this volume, you did not fulfill the obligation.



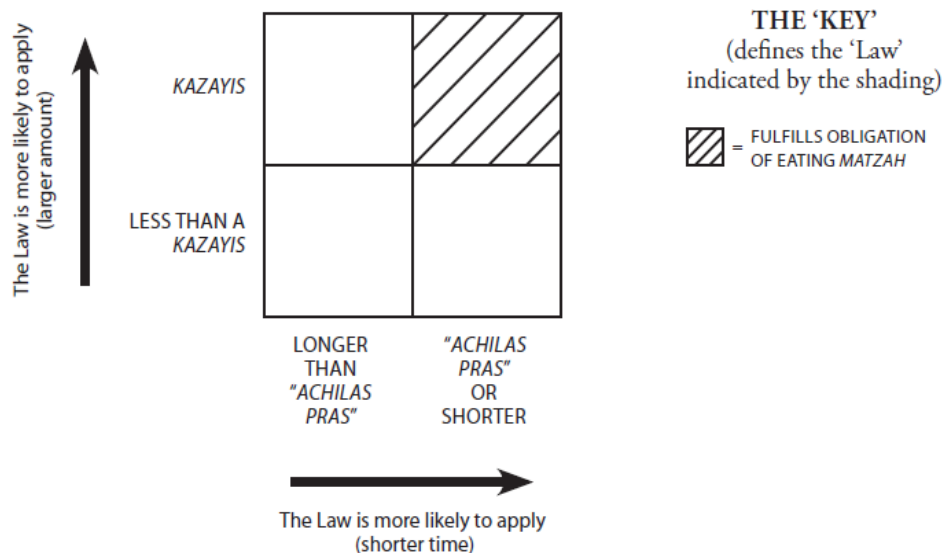
A Diagram that describes how much matzah must be eaten to fulfill one's obligation. The upper box represents a case where it is more likely that the obligation is fulfilled. Only the upper box is shaded to indicate that the obligation is fulfilled only if a kazayis is eaten.

The box that is not shaded indicates a situation in which the obligation is not fulfilled, whereas the shaded box indicates a situation in which a *kazayis* is eaten and thus the obligation is fulfilled. This representation is useful for any sort of legal statement: above a certain threshold, the Law applies and beneath it does not. The Key, represented by the arrow, tells us the direction (conventionally upwards) of the stringency of the Law. So let us say that the lower box was shaded and you were looking for the solution of the upper box, the solution is self-evident: it should be shaded too. Similarly, if the upper box was blank, you can deduce that the lower box is blank as well. This is the core of the method. Here, when only parameter is taken into account, the added value of the visual method is rather limited. It becomes worth the effort when two (or more) parameters come into the picture. We therefore move to the typical Talmudic Diagram, which is two-dimensional.

“The different shading patterns in a two by two Diagram tell us about the significance of the two factors as they relate to the Law in question.” (29)

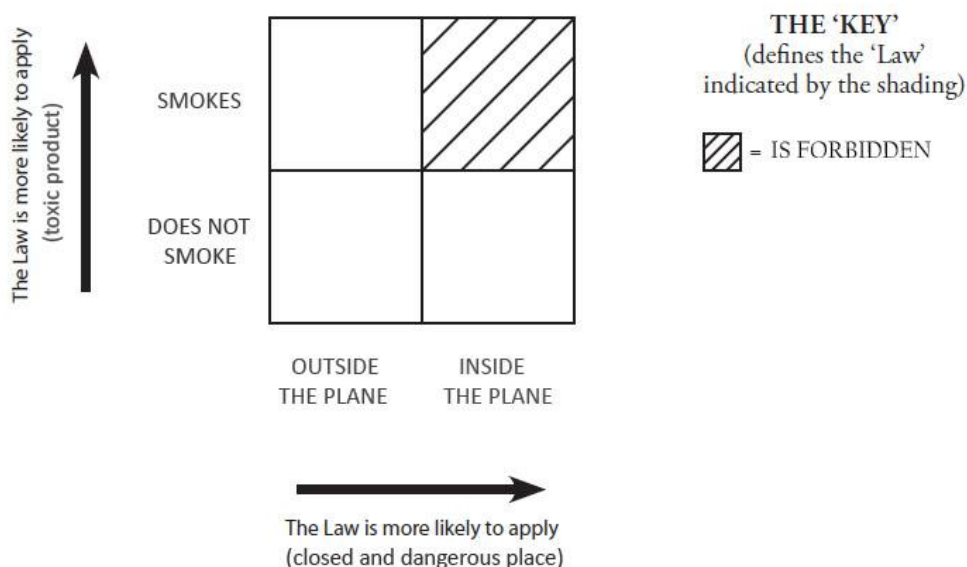
It includes two parameters, four boxes and two arrows. The arrows are conventionally directed upward and from left to right.

“A Talmud Diagram consists of rows and columns just like an ordinary table, but the boxes within the table each correspond to a specific ‘case’ within the Talmud. The rows and columns are arranged according to a specific plan dictated by the logic of the Talmud and the boxes formed at the intersection of the rows and columns are shaded to indicate whether a specific Law applies to that case or not. By examining a Diagram you can see at a glance to which cases that specific Law applies just by seeing if the box is shaded, or not.” (20)



The second parameter is the time it takes to eat half a loaf of bread. Thus, it is not enough to complete the obligation to eat a certain amount, but you also need to do this within a certain amount of time.

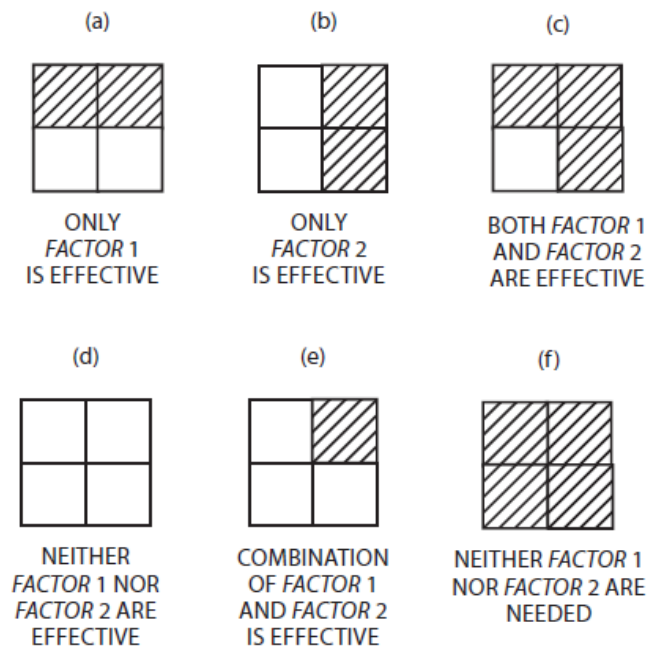
To take another example, if the driver drinks alcohol and takes drugs, and if either of them is forbidden, it is all the more forbidden to consume both. It could also be the case that each behavior is authorized by itself but it is forbidden to do both simultaneously. For example, it is not forbidden to smoke and it is not forbidden to take the plane, it is, however, forbidden to do both at the same time.



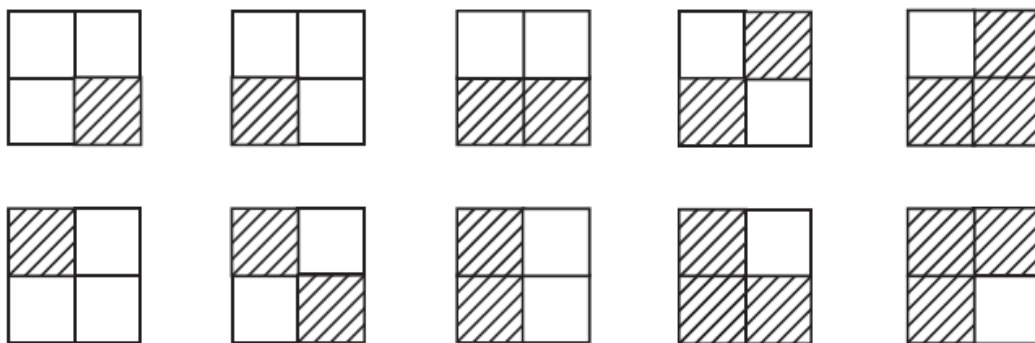
This is a valid Diagram. A Diagram is valid when it respects the Shading Rule:

"The Shading Rule: In a Diagram if a box is shaded, all boxes above it and to its right are also shaded. If a box is blank, all boxes below it and to its left are also blank". (27)

According to this rule, out of the sixteen possible patterns of two-dimensional Diagrams, only six ‘make sense’:

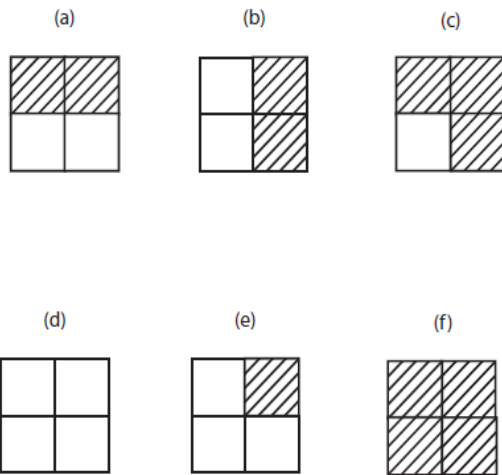


On the contrary, these ten Diagrams are not valid (30), as they do not respect the Shading Rule:



As the author puts it: “Theoretically there are sixteen possible patterns of shading for a two by two Diagram. Some of the patterns ‘make sense’ while others don’t. The six patterns shown in Figure 1.5 ‘make sense’ because they obey the Shading Rule, namely that whenever a box is shaded, all boxes above it and to its right are also shaded.” (30)

This distinction between valid and invalid Diagrams is very helpful to better visualize the relationships between necessary and sufficient conditions. Indeed, here is the wording of each valid Diagram. Let us say the vertical arrow bears on whether you speak Dutch (upper boxes) or you don’t (lower boxes). The horizontal arrow indicates whether you speak English (right boxes) or not (left boxes). You have four possibilities: you speak none of them, you speak Dutch but not English, English but not Dutch, you speak none of them. You can hereby visually represent the conditions to get a job:



- (a) It is necessary and sufficient to speak Dutch
- (b) It is necessary and sufficient to speak English
- (c) It is sufficient to speak either Dutch or English
- (d) It is not sufficient to speak Dutch and/or English
- (e) It is necessary and sufficient to speak both Dutch and English
- (f) It is not necessary to speak either Dutch or English

The Diagram method can also be used to represent disagreements. As you cannot possibly contend that your opinion is (represented by) one of the invalid Diagrams, you need to offer another view (literally) of the situation and a new – valid – Diagram. Once you have the two opposing Diagrams, you can see what the difference is. Usually the difference will depend upon. The main steps in the making of Diagrams are the following (pages 27 and 34):

- Step 1: “Create a Diagram that contains all the relevant cases, arranged in order of likelihood that the Law applies, with likelihood increasing up and to the right”;
- Step 2: “Using the style of shading shown in the Key, shade the boxes corresponding to cases where you know the Law applies. Leave boxes blank where you know the Law does not apply. Mark all remaining boxes with a question mark”;
- Step 3: “Use the Shading Rule to determine the shading status of as many of the remaining boxes as possible”;
- Step 4: “Create separate Diagrams for separate opinions”.

The four steps are summarized at the end of the book and accompanied by Diagrams (148-149). Two remarks are made about these steps (27). First, “**A Diagram with its pattern of shaded boxes represents an opinion**”: you should not represent two different opinions within the same Diagram but rather use one Diagram to represent each opinion. Second, “**The value of using Diagrams is to succinctly represent an opinion using a pattern of shaded boxes**”. The shaded boxes are a direct indicator of the opinions, but one should also pay attention to the fact that two opinions may differ only in the names of the variables but have the same boxes shaded. All the elements are therefore relevant.

The book is structured in a pedagogic way. It starts from simple cases and moves towards more difficult examples. You are asked questions to check if you properly understood the content of each chapter. The answers are at the end of the book. These are the chapters:

- Chapter 1: Constructing Diagrams
- Chapter 2: Disputes, Proofs and Refutations
- Chapter 3: The Language of Diagrams

- Chapter 4: Dealing with Time
- Chapter 5: Diagrams with Multiple Shadings
- Chapter 6: The *Kal Vachomer*
- Appendix: Three Dimensional Diagrams
- Answers to Questions
- The Four Steps

As is visible from the list of the chapters, it ends up with a chapter on the argument *a fortiori* (*kal vachomer*). This is how Ury defines this type of argument:

“The *kal vachomer* is a logical argument that proves a proposition to be true under one set of circumstances based on it being true under a less compelling set of circumstances.” (95)

The question of the link between the *kal vachomer* and the very technique of Talmudic Diagram is to be raised here. Is the *a fortiori* any sort of argument that the Talmudic Diagrams displays? Is it just one the many arguments that are visualized here? The answer is negative. The argument *a fortiori* rather seems to be not only part and parcel of Talmudic Diagram but is even the central if not unique argument that the methods depends on. These boxes are indeed organized in such a manner to deduce cases through the argument *a fortiori*: if the lower is shaded, the upper box must certainly be shaded, if the upper case is blank, the lower case must certainly be blank. Ury states:

“The principle of the *kal vachomer* is built into the fabric of Diagrams. Every time we completed a Diagram we did so using the principle of the *kal vachomer*, and the Shading Rule itself can be viewed as nothing more than a restatement of the principle of the *kal vachomer*.” (95)

The whole method is using the argument *a fortiori* visually to resolve cases of uncertainty. Whenever a box is left with a question mark and its status is to be determined, either an argument *a fortiori* leads to its solution or no solution at all is found. If no solution is found, it is no problem at all: we know that we don’t have the information to answer the case. In other words the argument *a fortiori* solves all the problems that can be solved. The other problems remain unsolved.

Two additional remarks on the argument *a fortiori*. First, although the whole method is founded on the argument *a fortiori*, the book does not enter all the technicalities of the device: “Notably missing in this chapter are the important concepts of *dayo* and *tzad hashaveh*” (95). Maybe in a future publication, Ury will tackle these problems.

Second, there is an interesting comment upon the argument *a fortiori*. The author says that one should not reduce this argument to a purely logical one. This could mean different things. It could mean that the argument *a fortiori* implies linguistic features such as scalarity, i.e. the interrelation between concepts. For example, if a drink is hot, it is at least warm. You could also say: the drink is warm *or even* hot. But you could not possibly (at least not easily) say: *the drink is hot *or even* warm. The reason why some statements are acceptable and some are not lies in the fact that words in natural languages, including English, are oriented: “warm” is oriented towards “hot”. The keywords that link these words are *at least*, *or even* and the like. To sum up, to say that the argument *a fortiori* is not purely logical could mean that it requires those keywords that are typical of natural languages. This happens to be our opinion. The other finds another reason why the argument *a fortiori* is not purely logical:

“The *kal vachomer* is valid because it is one of the Thirteen Hermeneutic Principles by which Torah Law is derived, not simply because of ‘logic’. The *kal vachomer* is distinct, for example, from the logical certainty of *Bichlal Ma’asaim Maneh* – the principle that states that 200 contains 100.” (95)

Of course, maybe the author would say this about the *kal vachomer* and not about a mere argument *a fortiori*.

It is important to recall the disclaimer of the author:

“There is no intention here to replace an understanding of the *sugya* with the manipulation of Diagrams. Rather, we use Diagrams and the language of Diagrams to follow and remember the workings of a *sugya*.” (47)

The method is not made to replace but to complement the study of the Talmud. It is our claim that it works very well. By the way and independently of the hypothetical structural difference between the argument *a fortiori* and the *kal vachomer*, the method made up in this book is extremely useful in general legal argumentation and not only in Talmudic discussions. One would do well to deepen the comparison between legal argumentation in Continental law and Common law and that in Talmudic argumentation. This book surely helps the reader advance on this topic, too.

Notes:

1. I would like to thank Jennifer Nigri for having helped me with the graphs and Tal Binyamin Polon for his comments.
2. I stick to Ury's capitalizing words like Diagram, Key or Law.
3. I stick to Ury's transcriptions of Hebrew.