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Dynamic Essences: Absolute, Prospective, Retrospective, and Relative Modalities

Błażej Skrzypulec

Polish Academy of Sciences,
Warsaw, Poland

e-mail: blazej.skrzypulec@gmail.com

Paweł Rojek

Jagiellonian University,
Krakow, Poland

e-mail: pawel.rojek@uj.edu.pl

Abstract:
Essential properties are usually thought as properties that things must always possess, whereas accidental properties are considered as changeable. In this paper, we challenge this traditional view. We argue that in some important cases, such as social or biological development, we face not only the change of accidents, but also the change of essences. To analyze this kind of change we propose an alternative view on the relations between the modalities and time. Some properties might be necessary or possible for a thing in a classical sense throughout its existence, whereas others might be necessary or possible only for some restricted periods. We distinguish therefore absolute, prospective, retrospective, and relative modalities. As we argue, these non-classical concepts of modality are useful in analysis of some puzzling case of seemingly changing essences.
Keywords: essentialism, modality, necessity, essential change.

1. Introduction

Usually essences are thought to be necessary and thus unchangeable. Essential properties of things are considered to be properties that things must possess, whereas accidental properties might or might not be possessed. The modal status of properties entails their relation to time. Necessary properties are properties that things always have, whereas contingent properties might change over time. Things can gain some new accidents, and can lose some old ones, but their essences remain the same. We would like to challenge this traditional view. We believe that in some cases it is possible to speak not only about changing accidents, but also about changing essences. It seems that in such phenomena as development or decline things might change their modal properties. Something that was possible in one stage of a process might become impossible in another, and conversely, something necessary at one time might turn out to be contingent in another. In other words, in some cases essences might turn into accidents and accidents might become essences. The standard view on the relations between essences, modalities, and time, which excludes such transformation, should therefore be revised.

It is clear that the classical concept of essence involves permanence in time. Though Aristotle's notorious expression denoting essence *to ti en einai* literally means "the what it *was* to be", it was generally understood as "the what it *is* to be", where "is" was thought to be timeless [14]. Essence was therefore traditionally thought as a property or properties belonging to a thing during all its existence. If P is essential property of *x*, then *x* cannot gain and lose P, but must possess it all the time it exists. The opposite, however, does not hold. Some accidents might belong to a thing for all its time, but this does not make them essences. Porphyry in his commentary on Aristotle's *Categories* gave famous examples: being black, for ravens, and risibility, for a human being [16, p. 12]. These properties, according to classical view, were not essential, even if were possessed by ravens and man throughout their whole life. Permanence was therefore thought as necessary, though not sufficient condition of essentiality.

This kind of link between essentiality and permanence has been strengthened by modern modalism, that is a view connecting essentiality with necessity. In this account essential properties are identified with those which are necessary for an object. The concept of necessity even more overtly involves permanence. If P is necessary property of *x*, then obviously *x* possesses it whenever exists. Modalism has been famously criticized by Kit Fine [8]. He argued that necessity and essentiality differ not only in their intension, but also extensionally: there are necessary properties which can hardly be recognized as essential ones. Mathematical necessary truths for instance are formally entailed by the existence of Socrates, but do not constitute his essence in any reasonable sense. Nevertheless, Fine and his followers, though argued that necessity is not a sufficient condition of essentiality, have never doubted that it is its necessary condition (see [8, p. 4]; [22, p. 211]. Being necessary does not entail being essential, but essentiality entails necessity. Since classical necessity entails permanence in time, it means again that essences are permanent.

Indeed, it seems plausible that permanent or necessary (in classical sense) properties of things are not always essential for them. The dogma, which we want to challenge here, however says the opposite, namely that essential properties are always permanent or necessary (in classical sense). This claim seems to be shared by all debating parties. We believe that the criticism of the modalism should be extended. Modalism claims that classical necessity is both sufficient and necessary condition of essentiality. It critics argued that it is not sufficient, we believe that it is also unnecessary. In our view, the concept of essence might involve many different kinds of necessity, not the classical one. Loosening of the link between essence and necessity makes a space for the concept of changing nature. Therefore, in this paper we would like to sketch a conceptual framework for new combinations of essentiality and time. We would like to distinguish a few possible concepts of essences. Some properties might be necessary for a thing throughout its existence, whereas others might be necessary only for some periods. Thus, apart from classical absolute essences there are also non-classical time-relative types. It seems that the traditional view is only a particular case of a more general and more dynamic stance. It turns out that the concept of essence might be connected with many different concepts of necessity. The classical necessity, i.e. entailing permanence, is not necessary for being essential. It is not only insufficient, as was argued by critics of modalism, but also unnecessary condition for essentiality.

First, in Section 2, we would like to point out some puzzling examples from various fields, which highlight the need to reconsider the standard view on essences. The simplest case is taken from sociology; more complicated cases are borrowed from theology and biology. All these case pose obvious difficulties for a classical views which do not allow changing essences. These examples, as we argue, cannot be also easily discussed in a simple framework of possible worlds. Second, in order to analyze these puzzling cases, we distinguish in Sections 3 and 4 four concepts of modality: absolute, prospective, retrospective, and relative, which lead to various concepts of essence. We define these concepts using a logical framework generally inspired by a branching approach to time and modality (see [21], [13] for general overview). We do not however stick to any particular version of this well-developed theory. Rather, we use some of its concepts and intuitions to construct a very simple framework needed to our purposes. Finally, in Section 5, we attempt to use that framework to interpret non-classical cases discussed in Section 2. We believe that the proposed temporal extension of classical essentialism might help in clarifying intuitions concerning modalities changing over time.

2. Three Modal Puzzles

Now we would like to introduce a few examples that show that in some cases it is plausible to speak about changing essences. Things can undergo deep ontological changes, which are much more radical than simple accidental modification, but this does not lead to the destruction of these things. This kind of change is neglected in the classical view.

2.1. Internalization

The first and the simplest example comes from sociology. Peter Berger and Thomas Luckmann investigated the complex process of the construction of social reality. Briefly, it consists of three fundamental stages: externalization, objectification, and internalization. People constantly define their own reality, afterwards treat it as independent, and finally transmit it to their children. The way people act is fundamentally contingent. We could choose, fix, and transmit completely different ways of behaving. Nevertheless, in the process of internalization the results of occasional human activity obtain the independent status of inevitability:

> The child does not internalize the world . . . as one of many possible worlds. He internalizes it as the world, the only existent and only conceivable world, the world *tout court*. . . . Primary socialization thus accomplishes what (in hindsight, of course) may be seen as the most important confidence trick that society plays on the individual – to make appear as necessity what is in fact a bundle of contingencies [3, pp. 154-5].

In other words, in the process of internalization some accidental and external characteristics, such as those that are default ways of acting in a given society, become essential and internal for individuals. If we were born and brought up in a different society, we would think and act in different ways, but once we successfully pass through the process of primary internalization in a determined society, we treat some particular ways of thinking and acting as fairly natural and in fact necessary.

Berger and Luckmann argue that primary socialization is in principle irreversible. The world internalized in this process is so deeply entrenched in consciousness that one cannot simply abandon it or distance oneself from it:

> Our analysis suggests that such distance is only possible with regard to realities internalized in secondary socialization. If it extends to the realities internalized in primary socialization, we are in the domain of what American psychiatry calls "psychopathy," which implies a deficient formation of identity [3, p. 230].

This means that the image of the world received in early childhood constitutes the very essence of human identity. The results of secondary socialization in school or a place of work are perceived as much more accidental. One can learn new things or change social roles without undermining one's fundamental sense of reality.

This sociological example of changing essence differs from classical cases, not only – as we suggested – because the essence changes in this case, but also because it is somehow subjective. Social reality depends on the definitions accepted by the members of a society. To be real in a social world is to be taken as real. Moreover, the loss of acquired essence does not literally lead to the cessation of the individual existence. The loss of the primary image of the world presumably leads to psychopathy, but not necessarily to suicide. This is the exact the point of difference between the sociological example and the two following cases taken from theology and biology, where essences are thought to be perfectly objective.

2. 2. Augustine

The second example is the most ancient and venerable, since it comes from St Augustine. He suggests in a few passages that the religious development of humankind, starting with creation of human beings and ending in their salvation, might be seen as a series of transformations of human nature. In the first place, humankind in Eden could do things that would be impossible for them in the final stage, in heaven. Augustine writes:

> The first freedom of the will was therefore to be able not to sin; the final freedom will be much greater: not to be able to sin. The first immortality was to be able not to die; the final immortality will be much greater: not to be able to die. The first power of perseverance was to be able not to abandon the good; the final happiness of perseverance will be not to be able to abandon the good. The final goods will be better and more powerful [1, p. 214].

According to Augustine, at the beginning of creation human beings could sin or not sin (*posse peccare et posse non peccare*), whereas at the end of salvation humankind will not be able to sin (*non posse peccare*).

Augustine used this idea of modal transformation to clarify the sense of the Fall and the Redemption. Due to Adam's original sin humankind lost the possibility of not sinning (*posse non peccare*) and was left with the mere possibility of sinning (*posse peccare*). After the Fall human beings could not *not* sin. The Redemption fortunately restored this corrupted human nature. Due to Christ's actions, human beings again acquired the possibility of not sinning (*posse non peccare*). Finally, our future Salvation will consist in the last irreversible modal change, which will exclude the possibility of sin [1, pp. 213-5].

Regardless of the subtlety of these theological matters, it seems that, for Augustine, human nature is substantially changeable. It allowed some possibilities at first that afterwards were apparently excluded. The essence of a human being was therefore thought to be dynamic, not static. This view obviously calls for a revision of the classical concept of essence.

2.3. Jellyfish

A similar example of a changing nature might be found in biology. We would like to focus on the lifecycle of a jellyfish. Most jellyfish start their existence in a larval form, after some time transform into a stationary polyp, and later undergo the final transformation, which results in an adult medusa. Such a description suggests the presence of some modal properties. First, it seems that a jellyfish has to possess the property of "being a larva" for some time at the beginning of its life. Second, it is possible for a jellyfish to stop being a larva, while still continuing its existence as a polyp and, after some time spent in a polyp stage, it is also possible for it to stop being a polyp and become an adult medusa. Third, after reaching the adult stage it is no longer possible for a jellyfish to stop being an adult medusa and yet continue its existence.

However, there are exceptions form the above "standard" pattern:

> [A] unique case of ontogeny reversal has been reported by Bavestrello et al. (1992), in which newly released, sexually immature medusae of *Turritopsis nutricula* McCrady, 1859, regressed, settled onto a substrate, and gave rise to stolons and hydroid colonies [15, p. 302].

It turns out that exemplars of *Turritopsis nutricula* jellyfish are able to return to the polyp stage even after reaching adulthood. Because the cycle of being an adult medusa and reverting to a polyp can – at least in some perfect environmental conditions – go on *ad infinitum*, the *Turritopsis nutricula* jellyfish does not have a limited life span. Within this peculiar life cycle it is no longer impossible to continue existence despite losing the property of being an adult medusa, due to the fact that existence may be continued in a form of a polyp.

This biological example shows that variations in modal properties occur both within the life cycle of a single jellyfish and between the life cycles of jellyfish belonging to different species. Jellyfish with "standard" life cycles may acquire the modal property of being an adult medusa, which then cannot be lost as long as the jellyfish lives. Because of this, their modal properties can change during their lifetime. The property of being an adult medusa does not have the same modal status in the life cycle of *Turritopsis nutricula*, as these jellyfish can live after losing this property. Because of this, we may speak about modal differences between the life cycles of various species. Modal properties like "being a larva", "being a polyp", and "being an adult medusa" seems to be good candidates for essential properties as they determine what an entity is at different stages of its development [6], [7]. However, their status cannot be characterized within a theory that only allows for essential properties that have to be possessed at all moments of an object's existence.

Now, it seems that all these examples challenge the classical essentialism. Apparently in some cases things can change their essences. During the process of ontological development things lose some former possibilities and gain new ones. As we saw, this process might be either irreversible, as in the cases of primary socialization, final salvation, and adultness of ordinary jellyfish, or reversible, as in the cases of secondary socialization, original sin, and becoming an adult *Turritopsis nutricula*. These processes can hardly be described as merely accidental changes. We find it perfectly plausible to say that these things change their modal properties, so they also change their essences. Something that was possible or necessary at one stage becomes necessary or possible in another. This is exactly what might be called a change of essence.

3. Four Kinds of Modal Properties

Now we would like to sketch a general conceptual framework for expressing the modal intuition of dynamic essences. First of all we would like to generalize the previous puzzling examples and distinguish a few kinds of modal properties. Because of that, in the next section, we shall propose a conceptual scheme inspired by a branching approach to time.

Let us consider some episodes from the life of the entity named "Kant". The existence of Kant started at a moment t_0. At that time Kant was identical to an embryo and so did not possess a brain. However, this situation only lasted until a later moment t_A, at which Kant developed a human nervous system. Let us focus first on two of Kant's properties: the property of having a brain and the property of being an embryo.

Having a brain can plausibly be considered an essential property of Kant. First, after moment t_A one important answer to the question "What is Kant?" consists in stating that Kant is a brain-possessing creature. Second, having a brain determines much of the Kant's other properties, and facts about Kant's brain may be used in explaining his actions [5]. Third, and the most important, having a brain seems to involve some modal aspect. While Kant does not possess brain at every period of his life, after moment t_A it is impossible for Kant to lose his brain without ceasing to exist (it is a so-called "phasal property"; see [2], [10]).

Being an embryo also seems to be an essential property of Kant (see [6, p. 188] for a similar example). It also determines what Kant was in the early stages of his development. Similarly to the case of having a brain, being an embryo is not a characteristic that Kant possessed for his whole life: at a certain moment t_B Kant lost this property. Despite this, the property of being an embryo also possesses a modal component. It seems that at moments earlier than t_B it was impossible for Kant to not be an embryo.

These two examples suggest that Kant both gained and lost essential properties during his lifetime: at t_A he gained the property of having a brain, while at t_B he lost the property of being an embryo. We may also consider other properties, which could be possessed for some time and in some broad sense might be called essential. For instance, being a philosopher is neither a property that Kant had at all moment of his life, nor a property that could not be lost after obtaining it. In such a case, is there a sense in which being a philosopher may be an essential, and so a necessary property of Kant? We may imagine that for Kant being a philosopher, a property gained by him at some moment t_C, was a deeply internalized aspect of his personality, which could not be lost in a short period of time, but only due to a lengthy process in which Kant's personality would be gradually transformed (see [19] for a similar intuition). If this is right, then we can state that it would have been impossible for Kant to stop being a philosopher during a certain, finite period of time after t_C. In other words, all temporally shorter ways of losing the property of being a philosopher would have led to the end of Kant's existence.

Of course, apart from the above three peculiar kinds of essential properties that can be gained or lost (or both) in time, Kant may be also described as possessing more standard ones. Probably being a *homo sapiens* serves as a plausible example of an essential property that Kant possessed at every moment of his existence, which he could not exist without.

It seems therefore that there are different types of essential properties, some of which may be gained or lost during the life history of an object. All these essential properties involve a modal component, since it may be stated that they are in some way necessary for an object that possess them.

It should be noted that the further considerations do not rely on our accepting the story about Kant as entirely true. One may doubt whether Kant really existed before the development of his brain or whether it is possible to internalize the role of philosopher so strongly that it cannot be rapidly lost. What is important is to observe that somebody may rationally accept the above story about Kant and his essential properties. Thus, we need a theory to explain the meaning of statements attributing different types of essentiality, and so different types of necessity, to Kant's properties.

Of course, one may simply reject the above problem by stating that the properties that an object cannot lose but can lack at some periods of its life, like "having a brain", are not necessary properties and so are not essential [18]. From this perspective only properties that an object cannot lack, like "being a *homo sapiens*", deserve the status of being essential. While such position is internally coherent, we believe that it is misguided from a methodological point of view. According to our pre-theoretical intuitions objects may have some special properties that determine what a given object is and are such that object in some sense "has to" possess them. These properties may be called "essential" and the role of a philosophical theory is to explain more precisely what the essentiality of properties means in accordance with basic intuitions. It seems to us that properties like "having a brain" in the story about Kant can be intuitively regarded as essential.

4. Modal Histories Framework

Now we would like to propose a simple formal framework for dynamic essences. First, we will introduce two sets – one representing time, and one representing qualities – then we shall combine them to arrive at the concept of a modal history of an object, which serves as a basis for further definitions of various kinds of necessities determining different types of essential properties.

The first set, $T = \{\ldots, t_0, t_1, \ldots\}$, is an infinite set of moments linearly ordered by the asymmetric (and so irreflexive) but transitive relation *is later than*. In addition, moments t_i and t_k are *successors* iff t_k is later than t_i, but there is no moment later than t_i and earlier than t_k.

The second set, Q = {A, B, C,…}, is a nonempty set whose elements are maximal sets of properties (MSP), excluding properties concerning an object's existence at a particular moment (e.g., "exists at t_1"). A set of properties is maximal if and only if for any property F, either F or ~F belongs to the set.

A Cartesian product Q × T is a nonempty set of *MSP at times*: QT = {…,<A,t_1>, <B,t_2>,…}. Now, by referring to the set QT, the crucial notion of the proposed framework, modal history of an object *x,* may be characterized.

The modal history of an object *x,* MH(*x*), is a structure composed of *MSP at times*, containing all *MSP at times* that *x* can have during its existence and only those *MSP at times*. For example, if <A, t_1> does not belong to the MH(*x*), then the entity *x* cannot exist at moment t_1 in a way characterized by A. Further, in the context of modal histories *MSP at times* will be called *points of a modal history*.

Because a modal history is a structure, there is a relation organizing the points of a modal history. More precisely, a relation is needed that describes how the properties of an object can change in subsequent moments. This relation cannot simply be the *is later than* relation connecting moments, as in this case all points containing earlier moments would be connected with all points that contain later moments. Such a solution wrongly excludes modal histories which, for example, includes points <A, t_1>, <B, t_1>, and <C, t_2>, but in which an object can be as it is characterized by C at t_2 only if at the previous moment it possessed properties included in A (and not those included in B).

We propose the introduction of an asymmetric and intransitive relation of *modal binding* that may connect points containing subsequent moments. If some points <A, t_1> and <B, t_2> stand in such a relations, it means that if an object possesses A-properties at t_1, then at t_2 it can possess B-properties. What is more, we may define that a point *k* of a modal history is in the asymmetric and transitive relation of *being further* (<) than point *g* of this modal history iff there is a chain of *modally bounded* points whose first element is *g* and last is *k*. If one point is further than another, then there is a pattern of changes that can lead from properties possessed at the earlier point to properties possessed at the further point.

A modal history can have a branching shape. Let us consider a very simple example of such a history (lines represent *modal binding* relations) (Fig. 1):

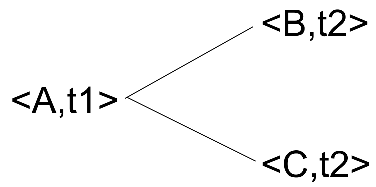

Fig. 1 A simple branching modal history.

According to the above diagram, an object can only exist at two moments: t_1 and a successive moment t_2. At moment t_1 it can exist if and only if it has properties belonging to the maximal set A. However, at t_2 it can exist in two different ways: having properties belonging to the maximal set B or having properties belonging to the maximal set C.

Up to this point, the framework of modal histories may seem analogous to models of branching-time, which describe tree-like structures composed of moments ordered by a *is later than* relation [21], [13]. Indeed, similarly to the branching-time approach, the properties of the *R* relation and the ordering of moments in the set T forbid backward structures (e.g., in which t_1 is later than t_2) and reflexive structures (e.g., in which t_2 is later from itself).

However, branching-time models usually put additional restrictions on the permitted structures. Most notably, in standard branching models structures can branch only towards the future, but not into the past. More formally, it is assumed that if t_k is later than t_i and t_k is later than t_j, then t_i=t_j or t_j is later than t_i, or t_i is later than t_j. Because of this, the structures described in branching-time models may intuitively be called "tree-like", or described as possessing many "branches" resulting from a single "trunk". However, if we look for a framework describing the ways in which an object can be at earlier and later moments, an analogous constraint should not be postulated in the case of modal histories. Let us consider another very simple modal history (Fig. 2):

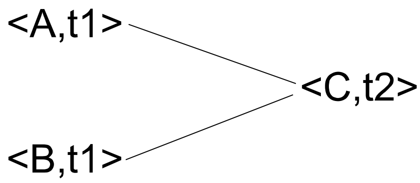

Fig. 2 A modal history that branches towards the past.

According to that diagram, an object at t_1 can exist as having properties belonging to the maximal set A or as having properties belonging to the maximal set B. However, in the successive moment t_2 it can exist only as having properties belonging to the maximal set C. The presence of such “modal bottlenecks” cannot be *a priori* rejected; in fact they may be quite popular, and so modal histories that branch towards the past should be permitted. Metaphorically speaking, modal histories often do not resemble well-groomed trees, but rather the rhizomes beloved of postmodern thinkers.

In addition, in the characterization of modal histories it is not even assumed that for any two points of the history, g_1 and g_2, it is the case that $g_1<g_2$ or $g_2<g_1$. In other words, a single modal history may be composed of unconnected “branches”. Let us consider a simple modal history once more (Fig. 3):

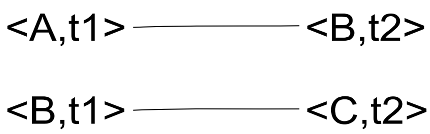

Fig. 3 A modal history compose of two unconnected branches.

An object with the modal history illustrated by the above diagram can exist at t_1 as having properties belonging to the maximal set A or as having properties belonging to the maximal set B. What is more, at t_2 it can exist as having properties belonging to the maximal set B or as having properties belonging to the maximal set C. However, if it has properties belonging to the maximal set A at t_1, than at t_2 it can only has properties belonging to the maximal set C, and if at t_1 it has properties belonging to the maximal set B, then at t_2 it can only have properties belonging to the maximal set D. Again, it seems that there is no *a priori* reason to exclude objects with such modal histories and so histories composed of unconnected branches should be allowed (see [5, pp. 121-23] and the criticism in [11]).

Having characterized the notion of a modal history of an object *x*, we may now show how it can be used in explaining the difference in modal status of Kant’s various essential properties. The framework of modal histories allows us to express various intuitively true modal statements concerning Kant. For example, it seems plausible that it would have been possible for Kant to start his life with properties different to those that he actually possessed. In such a case, in Kant’s modal history there would be at least two minimal points that have the same moment but different MSP (see Fig. 2).

Some more extravagant modal claims concerning Kant correspond to some structures of modal histories. For instance, one may claim that it was possible for Kant to have been born in Berlin and that in this case his life would have been completely different (he was actually born in Königsberg). If this is the case, then Kant’s modal history is composed of at least two unconnected branches (see Fig. 3). The minimal point of one of these branches has MSP with “being born in Berlin” as its element, while the minimal point of the second one has MSP with “being born in Königsberg.

4. 1. Absolute Necessity

Let us now consider how the framework of modal histories may help in explicating statements concerning the necessity of essential properties. We start with a classical concept of necessity, then we shall define non-classical cases. The classical, absolute concept would be referred to as $\square^A$, the "absolute necessity".

As stated above, "being a *homo sapiens*" seems to be an essential property of Kant. What is more, this property was possessed by Kant at every moment of his life and it was impossible for him to exist while lacking this property. In terms of a modal histories framework it can be stated that at each point of Kant's modal history "being a *homo sapiens*" belongs to MSP related to that point, or simpler, that at each point of his modal history Kant possesses "being a *homo sapiens*". Because Kant's modal history contains all "MSP at times" that Kant could have during his existence, the above statement expresses the idea that there would have been no possibility of Kant existing without being a *homo sapiens.*

This type of necessity can be called "absolute necessity" and defined as follows:

(D1) *At any point* g_i *belonging modal history of an object* x *(MH(x)) it is absolutely necessary for an object* x *to possess a property F (*$\square^A F(g_i)$*) iff at every point belonging to MH(x) the object* x *possesses F.*

$$\forall_{g_i \in MH(x)} \left(\square^A F(g_i) \leftrightarrow \forall_{g_k \in MH(x)} F(g_k)\right)$$

Analogously, the notion of "absolute possibility" may be defined by stating that at some point it is absolutely possible for an object to posses F if and only it has F at some point of its modal history:

(D2) *At any point* g_i *belonging MH(x) it is absolutely possible for an object* x *to possess property F (*$\diamond^A F(g_i)$ *) iff there is a point belonging to MH(x) that* x *possesses F at this point.*

$$\forall_{g_i \in MH(x)} \left(\diamond^A F(g_i) \leftrightarrow \exists_{g_k \in MH(x)} F(g_k)\right)$$

As it is easy to see, by considering the above definitions, that if at some point of a modal history it is absolutely necessary for an object to possess F, then at this point it is also absolutely possible for an object to possess F. What is more, if at some point it is absolutely necessary for an object to possess F, then at every point of its modal history it is absolutely necessary to possess F. The same goes for absolute possibility: if it is absolutely possible to possess F at some point, then at all points it is absolutely possible to possess F.

In the case of Kant's modal history, at each point Kant possesses "being a *homo sapiens*" and so at each point it is absolutely necessary for him (and so also absolutely possible) to be a *homo sapiens*. Now we can easily see that the necessity of another of Kant's essential properties, "having a brain", cannot be absolute necessity. It is not the case that Kant possesses "having a brain" at every point of his modal history, since at some points, at least those corresponding to the actual early phase of his development, he lacks this attribute.

4.2. Prospective Necessity

There were some moments in the actual life of Kant at which he did not have a brain. What is more, there are possible histories of Kant's life in which his life ended very early such that he did not have a brain at all. The necessity of "having a brain" for Kant arises from the fact that after developing a brain it is no longer possible for Kant to lose a brain and continue to exist. While there may be problems with characterizing such necessity in terms of possible worlds, it can easily be done within the framework of modal histories. Kant possesses a brain in a necessary way at some point of his modal history, because at every further point he possesses the property of having a brain. This type of necessity can be called "prospective necessity" and defined as follows:

(D3) *At any point* g_i *belonging to MH(x) it is prospectively necessary for* x *to have a property F* ($\Box^{\rightarrow} F(g_i)$) *iff for every* g_k *belonging to MH(x), if* g_k *is further than* g_i, *then object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left(\Box^{\rightarrow} F(g_i) \leftrightarrow \forall_{g_k \in MH(x)} \left(g_i < g_k \rightarrow F(g_k)\right)\right)$$

The notion of prospective necessity is a counterpart of the temporal logic operator G ("It will always be the case that …", [17, p. 13]), where *Gp* is true at some moment if and only if *p* is true at all later moments.

Analogously, a notion of "prospective possibility" can be defined:

(D4) *At any point* g_i *belonging to MH(x) it is prospectively possible for* x *to have a property F* ($\Diamond^{\rightarrow} F(g_i)$) *iff there is* g_k *belonging to MH(x) such that* g_k *is further than* g_i *and object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left(\Diamond^{\rightarrow} F(g_i) \leftrightarrow \exists_{g_k \in MH(x)} \left(g_i < g_k \wedge F(g_k)\right)\right)$$

The above definitions entail that prospective possibility follows from prospective necessity.[1] In addition, if at some point it is prospectively necessary for an object to have F, then also it is prospectively necessary to have F at all further points.

If our modal intuitions about "having a brain" and Kant are correct, then in the modal history of Kant there is a point at which it is prospectively necessary, but not absolutely necessary, for him to have a brain. In fact, a stronger statement also seems plausible: that "having a brain" cannot be possessed by Kant in any weaker sense than that specified by prospective necessity. Speaking more precisely, if at some point g_i belonging to Kant's modal history Kant possesses "having a brain", then at g_i it is prospectively necessary for Kant to possess "having a brain". Perhaps there are more properties like having a brain, properties of which it is true that if they are possessed, they are possessed prospectively necessarily.

The notion of prospective necessity is weaker than absolute necessity. If at some point it is absolutely necessary to possess F, then at this point it is prospectively necessary to possess F, but not conversely. Because of this, at some points of Kant's modal history it can be prospectively necessary for him to possess a brain, while it may still be true that he does not have a brain at every point. While there is a form of necessity connected with the property of having a brain, it is a different type to that exemplified by the absolutely necessary "being a *homo sapiens*". "Having a brain" is a candidate for an essential property that can be gained during an object's history: an object cannot lose it, but it can lack this property at some points of its existence.

One may ask, whether the notion of prospective necessity, and subsequent notions of retrospective and relative necessities, can be expressed in the more usual framework of possible worlds. We believe that it can be done, in a certain version of such framework, but we prefer to use the proposed modal histories framework as it seems to rests on weaker assumptions. If a necessary property of an object is defined as a property that on object has in all possible worlds in which it exists [20], [23], then the notion of prospective necessity cannot be formulated. This point can be demonstrated by considering the property of having a brain. Unfortunately, it is not the case that Kant has a brain at some moment in every possible world in which he exists, because in some possible worlds he died in the very early stages of development. What is more, a weaker statement, according to which in each possible world where Kant exists longer than X there is a moment in which he has a brain, is also not true. It seems to be logically – and probably also physically –

possible to prolong the early brainless stage of Kant's development for an indefinite amount of time.

A more promising idea is to develop a "two-dimensional" possible worlds framework, in which properties are possessed not just in a given world but in a world at a given time. Then, it can be stated that a property F of an object *x* is prospectively necessary if and only if for every moment *t* in every world in which *x* exists, *x* has F at every moment later than *t*. However, such solution has an important drawback. The crucial idea of our paper is that an object can change its nature by changing the modal status of its properties. Unfortunately, the above solution does not leave a space for expressing that, for example, a property F is merely contingent for an object *x* at one moment but then starts to be prospectively necessary. It is so because while the definition of F's prospective necessity involves time it is not a definition of F's being prospectively necessary at a particular time.

To amend this problem another modification of possible worlds framework is needed, which introduces trans-world moments and an accessibility relation that connects certain worlds-times pairs (analogous to our "modal binding"). If some moments, like t_m and t_n where t_n is earlier than t_m, can belong to many possible worlds, then it can be stated that F is prospectively necessary for *x* at t_m in world *W* because in every possible world, accessible from t_m in world *W*, in which *x* and t_m exists, *x* has F at all moments later than t_m. Despite that F may be contingent for *x* at t_n in world *W* due to the fact that not in all worlds, accessible from t_n in world *W*, in which *x* and t_n exist, F is possessed by *x* at all moments later than t_n. From this perspective, every maximal branch of a modal history of *x* may be identified with a set of possible worlds which do not differ in respect of *x*'s properties through time. However, such possible worlds framework assumes not only the possibility of identifying objects between possible worlds, which is problematic on its own grounds, but also the possibility of moments trans-world identification. The possible histories framework developed in this paper does not need any of these and utilizes only an intuitive idea that an object's lifetime could have been different from the actual one.

4.3. Retrospective Necessity

The necessity of "being an embryo" is a mirror image of the necessity connected with "having a brain" (at least in the context of Kant's life). Neither of these properties was possessed by Kant at every moment of his actual life. However, while it was impossible for Kant to lose his brain, the same does not hold about the property of being an embryo. In fact, the situation is reversed: it is possible that Kant is an embryo at some moment t_i but does not have this property at later moments; but is it impossible that he is not an embryo at moments earlier than t_i.

In terms of the modal histories framework, we may state that at least at some moments of Kant's actual life it was "retrospectively necessary" for him to be an embryo, where retrospective necessity is defined as follows:

(D5) *At any point* g_i *belonging to MH(x) it is retrospectively necessary for* x *to have a property F* ($\Box^{\leftarrow}F(g_i)$) *iff for every* g_k *belonging to MH(x), if* g_i *is further than* g_k*, then object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left(\Box^{\leftarrow}F(g_i) \leftrightarrow \forall_{g_k \in MH(x)} \left(g_k < g_i \rightarrow F(g_k)\right)\right)$$

The notion of retrospective necessity is a counterpart of the temporal logic operator H ("It has always been the case that …", [17, p. 32]), where *Hp* is true at some moment if and only if *p* is true at all earlier moments.

Similarly to case of prospective modalities, the notion of "retrospective possibility" can be characterized:

(D6) *At any point* gi *belonging to MH(x) it is retrospectively possible for* x *to have a property F* ($\diamond^{\leftarrow} F(g_i)$) *iff there is* g_k *belonging to MH(x) such that* g_i *is further than* g_k *and object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left(\diamond^{\leftarrow} F(g_i) \leftrightarrow \exists_{g_k \in MH(x)} \left(g_k < g_i \wedge F(g_k)\right)\right)$$

Again, it can be easily noticed that retrospective necessity entails retrospective possibility.[2] What is more, as in the case of prospective necessity, retrospective necessity is weaker than absolute necessity. An object can possess a property in a retrospectively necessary way at some points its modal history without having this property at all points.

Going back to Kant's modal history, we should postulate that at some points, corresponding to certain stages of Kant's actual life, it was retrospectively necessary for him to be an embryo. In case of "having a brain" it is also plausible to assume that Kant could not possess this property without possessing it in a prospectively necessary way. We may ask if an analogous statement regarding "being an embryo" should also be accepted, i.e. whether it is true that, if at some point g_i belonging to Kant's modal history, Kant possesses "being an embryo", then at g_i it is retrospectively necessary for Kant to be an embryo.

It seems a little less intuitive to accept the above claim than its counterpart concerning "having a brain". This is because we may imagine that in some non-actual parts of Kant's modal history he develops beyond the embryonic stage but then, due to some science-fiction nanotechnology, is reversed to the earlier phase. If such scenarios are possible, then the modal (and so essential) status of "being an embryo" is not uniform across Kant's modal history and only in some parts of it is being an embryo possessed in a retrospectively necessary way.

The notion of retrospective necessity designates a third type of necessity, different from both absolute and prospective necessities, and so may be regarded as connected with yet another type of essential property. Such properties do not have to be possessed at every point of an object's modal history, but if they are possessed at some point, then they are possessed at all earlier points up to the starting moment of an object's existence. In other words, if an essential property is necessary in a prospective sense, it can be gained during the object's existence, but then cannot be lost before its end. Reversely, if an essential property is necessary in a retrospective sense, it can be lost during the object's existence, but the object could not exist without it at earlier moments.

4. 4. Relative Necessity

The kind of necessity that was connected with "being a philosopher" in Kant's life seems to be even weaker than prospective and retrospective necessities. The property of being a philosopher can not only be gained at some moment later then the starting moment of an object's existence, but can also be lost before an object ceases to exist. Why then should we postulate that possessing such a property is necessary in any sense? It is necessary if, as is claimed in the earlier story concerning Kant, after gaining this property an object has to possess it for some period of time. Further, we will refer to this weak type of necessity as "relative necessity".

In terms of the modal histories framework the above idea can be expressed by stating that at some point an object possesses a property in a relatively necessary way if and only if it has this property at all further points in some range. To characterize the notion of "relative necessity" more precisely, we will need to define a concept of the "upper-limiting set of points of MH(x)" and "lower-limiting set of points of MH(x)":

(D7) $G^{x}_{t_j UP}$ *is a upper-limiting set of points of a MH(x) if and only if elements of* $G^{x}_{t_j UP}$ *are all points of MH(x) whose second element is earlier or equal to* t_j *(e.g.* <A, t_j>, <B, t_{j-1}>*) and only those points*.

$G_{t_jLOW}^x$ is a lower-limiting set of points of a MH(x) if and only if elements of $G_{t_jLOW}^x$ are all points of MH(x) whose second element is later or equal to t_j (e.g. <A, t_j>, <B, t_{j+1}>) and only those points.

By using the notions presented in (D7), relative necessity can be defined. However, the situation is a bit more complicated, as there is more than one type of relative necessity. First, there is "prospective relative necessity", which occurs at some point of a modal history if and only if an object has to possess a property up to a certain *further* point. Second, we can distinguish "retrospective relative necessity", which occurs at some point of a modal history if and only if an object has to possess a property up to a certain *earlier* point. Third, both prospective and retrospective types of relative identity come in different versions connected with the temporal distance between a point at which it is relatively necessary to possess a property and the point up to which this property has to be possessed. Due to these complications we may provide two general definitions of "relative prospective necessity" and "relative retrospective necessity":

(D8) *At any point* g_i *belonging to MH(x) it is relatively prospectively necessary for* x *to have a property F ($\Box_R^{\rightarrow} F(g_i)$) iff there is $G_{t_jUP}^x$ such that if a point* g_k *belongs to $G_{t_jUP}^x$ and* g_k *is further than* g_i, *then object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left[\Box_R^{\rightarrow} F(g_i) \leftrightarrow \exists_{v \subset MH(x)} \left(G_{t_jUP}^x(v) \wedge \forall_{g_k}(g_k \in v \wedge g_i < g_k \rightarrow F(g_k))\right)\right]$$

(D9) *At any point* g_i *belonging to MH(x) it is relatively retrospectively necessary for* x *to have a property F ($\Box_R^{\leftarrow} F(g_i)$) iff there is $G_{t_jLOW}^x$ such that if a point* g_k *belongs to $G_{t_jLOW}^x$ and* g_k *is earlier than* g_i, *then object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left[\Box_R^{\rightarrow} F(g_i) \leftrightarrow \exists_{v \subset MH(x)} \left(G_{t_jLOW}^x(v) \wedge \forall_{g_k}(g_k \in v \wedge g_i > g_k \rightarrow F(g_k))\right)\right]$$

Of course, two corresponding notions of "relative prospective possibility" and "relative retrospective possibility" may also be defined:

(D10) *At any point* g_i *belonging to MH(x), it is relatively prospectively possible for* x *to have a property F ($\Diamond_R^{\rightarrow} F(g_i)$) iff there is $G_{t_jUP}^x$ such that there is a point* g_k *that belongs to $G_{t_jUP}^x$ and* g_k *is further than* g_i *and object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left[\Diamond_R^{\rightarrow} F(g_i) \leftrightarrow \exists_{v \subset MH(x)} \left(G_{t_jUP}^x(v) \wedge \exists_{g_k}(g_k \in v \wedge g_i < g_k \wedge F(g_k))\right)\right]$$

(D11) *At any point* g_i *belonging to MH(x) it is relatively retrospectively possible for* x *to have a property F ($\Diamond_R^{\leftarrow} F(g_i)$) iff there is $G_{t_jLOW}^x$ such that there is a point* g_k *that belongs to $G_{t_jLOW}^x$ and* g_k *is earlier than* g_i *and object* x *possesses F at* g_k.

$$\forall_{g_i \in MH(x)} \left[\Diamond_R^{\rightarrow} F(g_i) \leftrightarrow \exists_{v \subset MH(x)} \left(G_{t_jLOW}^x(v) \wedge \exists_{g_k}(g_k \in v \wedge g_i > g_k \wedge F(g_k))\right)\right]$$

Analogously, as in the case with other types of necessity, here relative possibilities are also entailed by respective relative necessities. What is more, the relative necessity is the weakest form of necessity. First, if at some point it is absolutely necessary to possess a property, then at this point it is both prospectively and retrospectively relatively necessary to possess this property. Second, possessing a property in a prospectively necessary way entails that it is possessed relatively prospectively necessary. Finally, possessing a property in a retrospectively necessary way entails that it is possessed relatively retrospectively necessary.

If the earlier story about Kant's life is true, then at some points of his modal history it is relatively prospectively necessary for him to be a philosopher. However, it is very unlikely that at each point at which he is a philosopher it is relatively prospectively necessary for him to possess this property. In this case "being a philosopher" may be relatively necessary at some parts of Kant's modal history, but at other parts this property may not be connected even with this weakest type of necessity.

So far we have characterized four general variants of necessity, which may correspond with four distinct types of essential properties. Essential properties that are absolutely necessary are possessed by an object at every point of its modal history. In Kant's case, "being a *homo sapiens*" seems to be a legitimate candidate. Prospective necessity is connected with essential properties that can be gained during an object's existence, but then cannot be lost up to its end. It seems plausible that if Kant has a brain at some point in his modal history, then at this point it is prospectively necessary for him to possess a brain. Retrospective necessity is a mirror image of prospective necessity. It is connected with essential properties that can be lost at some point of existence, but nevertheless have to be possessed at all earlier moments. We argued that there are some points in Kant's modal history at which "being an embryo" is for him necessary in a retrospective way. Finally, relative necessity designates a class of essential properties that may be gained at some point and then lost at a later time. Despite this, they may be characterized as necessary because they display a "modal inertia". For example, in case of relative prospective necessity, they cannot be lost for some period of time. It may be the case that "being a philosopher" is relatively necessary for Kant at some points in his modal history.

5. Puzzles Reconsidered

Now we are ready to turn back to the examples introduced at the beginning of this paper. approach. We believe that the above-proposed conceptual framework may help to clarify these puzzling cases of dynamic essences in sociology, theology, and biology.

5.1. Social Internalization

The simplest case is the process of social internalization. During primary socialization a socially constructed image of the world becomes a part of individual identity. One cannot lose this without losing one's own personal integrity. That is why in the case of internalization it can be said that some accidental social properties become essential for individual human beings.

This process can be simply characterized with a help of the introduced concepts. As is clear from the discussion on Kant, being human involves a complex combination of different kinds of modalities. First of all, all humans presumably have some absolute necessary properties. Perhaps being a material substance or being a rational animal are examples of such properties. These constitute what is called 'nature' in classical essentialism. We might denote such absolute necessary properties as $\{\Box^A P\}$.

Now, humans are, however, amazingly flexible entities. The same absolute nature might be joined with different cultural extensions. Thus, second, in the early stages of its, a human has many different prospective possibilities. A child could be raised in this or that culture, could internalize this or that image of the world, and could therefore act in this or that way. At the beginning the modal properties of humans, besides some absolute necessary properties P, also embrace many prospective possible properties Q: $\{\Diamond^{\rightarrow} Q\}$.

Suppose now that a child was raised in a determinate culture, and acquired its first language and internalized some primary world-view. After successful primary socialization, something has essentially changed. Now we have no more *tabula rasa*, but rather *tabula scripta*, at least partly. Some possibilities allowed in the first stage have been realized and now determine the modal status of the individual in a new way. According to Berger and Luckmann, as quoted above, the process of primary socialization is irreversible. Once one acquires a social identity, one cannot lost it without a crisis of identity and even mental pathology. It seems therefore that this is a case of prospective necessity: $\{\Box^{\rightarrow}R\}$.

Usually, however, the modal determination in not so all-embracing. After successful primary socialization a human being could choose many different forms of secondary socialization. A child can still become a firefighter, doctor, or philosopher, even though it cannot reverse the process through which it became a child of a determinate culture. These secondary social roles are important for one's identity, but not in such a deep existential way as one's primary image of the world. This is because they are perceived as accidental. Even the child feels that it could become somebody completely different. This means that a human being, after primary socialization, achieves a new set of prospective possibilities: $\{\diamond^{\rightarrow} S\}$.

The whole process of primary socialization might be therefore described as a transition from a one modal stage to another. It might be depicted in the following way:

$$\{\Box^{A}P, \diamond^{\rightarrow} Q\} \rightarrow \{\Box^{A}P, \Box^{\rightarrow}R, \diamond^{\rightarrow} S\}$$

The absolute essences remain unchanged; new prospective-essential properties are achieved; the prospective possibilities are accordingly changed. This is the same human who passed through primary socialization, but she acquires a new nature.

5. 2. The History of Salvation

Saving humankind is more complicated that raising a child. According to St. Augustine, human salvation does not consist in simple essentialization, as primary socialization does, but also in a series of modally relative essentializations and de-essentializations, which were not allowed in the former sociological case. Moreover, in this case we are faced with true objective modalities; psychological integrity is not at stake here, as in the previous case, but the very existence of an individual, just as in classical essentialism.

At the beginning everything was possible, leaving aside the presupposed absolute essence of humankind. Humans in Eden could sin or not sin. Perhaps the first human thought that these were prospective modalities, but they turned out to be relative only: $\{\diamond_{R}^{\rightarrow} S, \diamond_{R}^{\rightarrow} \neg S\}$.

This modal *status quo* changed after the first realization of the possibility of sinning. It turned out that the first sin was a modal trap. After the Fall, humankind could not *not* sin. If that were the end of the story, humankind would be eternally condemned to sinning. Again, afterwards it turned out this was not a prospective modality, which would exclude any form of salvation, but only a relative one: $\{\Box_{R}^{\rightarrow}S\}$, that is: $\{\neg \diamond_{R}^{\rightarrow} \neg S\}$.

The Redemption was apparently a reversion of this modal essentialization. Christ's resurrection restored the previous modal status of humankind. The difference between humankind before the Fall and humankind after the Redemption lies, however, not only in their previous experiences. Now humankind can again sin or not sin, but this time the realization of the possibility of sin does not, as it seems, lead to a modal trap. It is plausible then to replace the relative possibility of not sinning with prospective possibility: $\{\diamond_{R}^{\rightarrow} S, \diamond^{\rightarrow} \neg S\}$. After the Redemption, we always retain the possibility of making good things.

The final Salvation, according to Augustine, is the exclusion of the possibility of sin. It is something like the inversion of the Fall. After the Fall, humankind could not *not* sin, whereas after the Salvation it cannot sin. Salvation, therefore, is an essentialization of sancticity. It seems that this modal shift should be thought not as relative, but as prospective: $\{\Box^{\rightarrow}\neg S\}$, in other words: $\{\neg \diamond^{\rightarrow} S\}$.

Therefore Augustine's theological history of creation, the Fall, the Redemption, and Salvation of humankind is a complicated story of relative essentialization and de-essentialization of sins and virtues:

$\{\diamond_R^{\rightarrow} S, \diamond_R^{\rightarrow} \neg S\}$ → $\{\neg \diamond_R^{\rightarrow} \neg S\}$ → $\{\diamond_R^{\rightarrow} S, \diamond^{\rightarrow} \neg S\}$ → $\{\neg \diamond^{\rightarrow} S\}$

Its final result is the necessitation of the former mere possibility to not sin. This general process, however, was interrupted by the relative essentialization of sin and its prospective de-essentialization. It seems that the proposed conceptual framework might really be adopted to clarify these complicated matters.

5. 3. Jellyfish Life

Now we can turn to the most complicated case of *T. nutricula* jellyfish life. The description of a standard jellyfish life cycle, presented by Piraino et al. [15], suggest that the life of a jellyfish consists in three phases, during which its essential properties change. First, a jellyfish starts its life as a larva and stays in this form for a certain period of time. Second, it transforms from a larva to a polyp and also possesses this form for some time. Finally, it changes from polyp to an adult medusa. This final stage lasts till the end of the organism's life. However, the life cycle of *T. nutricula* seems to be special, since this jellyfish is able to revert from the adult stage to the polyp stage, and then again become an adult in a potentially infinite cycle.

Similarly to a "standard" jellyfish, *T. nutricula* starts its existence in larval form. This means that at all the minimal points of its modal history, it possesses "being a larva". What is more, it has to remain in a larval stage at a certain number of later moments. Because of this, at early points of a modal history, it is retrospectively necessary for *T. nutricula* to be a larva since it has this property at all earlier points up to the minimal ones. In addition, at these early points it is also relatively prospectively necessary to be a larva, due to the fact that this property cannot be lost for a certain period of time. We may state that in the early phases of life *T. nutricula* has the following set of essential properties: $\{\square^{A} G, \square^{\leftarrow} L, \square_{R}^{\rightarrow} L\}$, where L designates "being a larva" and G symbolizes all absolutely necessary properties which have to be possessed by *T. nutricula*.

However, it is not the case that at all points of a modal history the set of *T. nutricula*'s essential properties equals $\{\square^{A} G, \square^{\leftarrow} L, \square_{R}^{\rightarrow} L\}$. At some distance from the minimal point of a modal history, there are two points g_i and g_k such that g_k is a successor of g_i (i.e., they stand in a *modal binding* relation), and at g_i the jellyfish is a larva but at g_k it is a polyp. Such a situation has to occur within a modal history if it is possible for *T. nutricula* to transform from the larval stage into a polyp. Then, at point g_i it is no longer relatively prospectively necessary to be a larva, as at one of the successive moments the jellyfish is a polyp. Nevertheless, at g_i it is still retrospectively necessary to be a larva, since a jellyfish is a larva at all earlier moments. Because of this, the set of essential properties shrinks to: $\{\square^{A} G, \square^{\leftarrow} L\}$.

What is more, a set of essential properties undergoes another modification as soon as *T. nutricula* becomes a polyp. As was stated above, in the modal history of *T. nutricula* there is a point g_i at which the jellyfish is a larva and a successive point g_k at which it possesses "being a polyp". According to a biological story, a jellyfish has to be a polyp for some time after acquiring this property. This means that at point g_k it is relatively prospectively necessary for a jellyfish to be a polyp. Nevertheless, at this point it is still retrospectively necessary for it to be a larva, as a jellyfish is a larva at all earlier points. Because of this, between g_i and g_k the set of essential properties expands to the following form: $\{\square^{A} G, \square^{\leftarrow} L, \square_{R}^{\rightarrow} P\}$, where P designates "being a polyp".

The above stage is very short and the set of essential properties changes again just after point g_k. If at g_k it is relatively prospectively necessary to have "being a polyp", then at all successive points a jellyfish is a polyp. However, at these points it is no longer retrospectively necessary for a jellyfish to be a larva, because there is an earlier point, i.e. the point g_k, at which it is not a larva but a polyp. Due to this fact at points further than g_k the set of essential properties shrinks again to the form $\{\square^{A} G, \square_{R}^{\rightarrow} P\}$.

A jellyfish may transform once again during its lifetime, this time from a polyp to an adult medusa. If this is the case, then again in its modal history there is a point g_m at which it is a polyp and a successive point g_n at which it is an adult medusa. Points such as g_m designate another modification of essential properties. At g_m it is no longer relatively prospectively necessary to be a polyp, due to the presence of the successive point g_n. Because of this, only those properties that are necessary in an absolute way belong to the set of essential properties possessed at g_m: $\{\Box^A G\}$.

The further modification of the set of essential properties occurs at the first point at which a jellyfish is an adult medusa (such as the point g_n characterized above). In the case of a "standard" life cycle, a jellyfish has to possess the property of being an adult medusa up to the end of its life and so at each point at which a jellyfish possesses this property, it possesses it in a prospectively necessary way $\{\Box^{\rightarrow} M\}$ (M designates "being an adult medusa").

However, in the special life cycle of *T. nutricula*, the set of essential properties $\{\Box^A G, \Box_R^{\rightarrow} P\}$ can shrink to $\{\Box^A G\}$ just before the possibility of becoming an adult medusa arises, and then, if the property of being an adult medusa is acquired, change to $\{\Box^A G, \Box_R^{\rightarrow} M\}$, instead of $\{\Box^A G, \Box^{\rightarrow} M\}$, known from the "standard" life cycle. Further, when in a successive moment there is the possibility to return to the polyp stage, the set shrinks again to $\{\Box^A G\}$, and then, if the reversal from the adult stage to the polyp stage occurs, the set is again $\{\Box^A G, \Box_R^{\rightarrow} P\}$. While it is unlikely, it is possible for such a cycle to repeat infinitely in the life of a particular *T. nutricula*. Overall, the pattern of changes in the essential properties within the life of a *T. nutricula* can be presented as a sequence of sets that ends with a loop:

$$\{\Box^A G, \Box^{\leftarrow} L, \Box_R^{\rightarrow} L\} \rightarrow \{\Box^A G, \Box^{\leftarrow} L\} \rightarrow \{\Box^A G, \Box^{\leftarrow} L, \Box_R^{\rightarrow} P\} \rightarrow \{\Box^A G, \Box_R^{\rightarrow} P\}$$
$$\leftrightarrow \{\Box^A G,\} \leftrightarrow \{\Box^A G, \Box_R^{\rightarrow} M\}$$

The framework of modal histories used here thus makes possible an account of the changing essences of biological organisms. *T. nutricula* seems to both lose (e.g., "being a larva") and gain (e.g., "being a polyp") essential properties during its life.

6. Conclusion

In this paper we attempted to combine modality and time in a new way. Traditionally modalities are thought to be timeless. Classical necessities and possibilities hold for any time when an individual exists. We believe that such approach cannot do justice to the common phenomena of development.

Some changes really involve a modification of the modal status of a thing, but nevertheless do not lead to its destruction. It is, after all, the same child that was born and raised in a determinate culture, the same humankind that fell and was saved, and finally the same jellyfish that pass through all the stages of their life-cycle. All these changes involve a deep modal shift: some things that were possible become necessary, and *vice versa*. In other words, they are examples of real essential change. These cases, to our minds, challenge the traditional view of static essences.

We propose dissolving the close connection between modality and time and unite them in new ways. We distinguished four such ways: absolute, prospective, retrospective, and relative modalities. Classical cases turned out to be simply extreme points of a large range of modalities. We tried to show that such simple modifications make possible a clarification of some puzzling real examples from sociology, theology, and biology.

One common charge against classical essentialism is that it excludes the real development of things. Ancient static essences, it is said, are incompatible with the contemporary dynamic vision of the world. On the other hand, modern anti-essentialists are accused of neglecting the real modal constraints that determine the process of development. Clearly not everything might really become something else, or not always. We believe that both sides of this discussion are right and we hope that our investigation shows the way in which these two opposite views might be reconciled.

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Notes

1. The maximal points of a modal history, i.e. points after which there are no further points, constitute an exception. According to (D3) and (D4), at maximal points everything is prospectively necessary but nothing is prospectively possible.
2. The minimal points of a modal history, i.e. points that have no earlier points, constitute an exception. According to (D5) and (D6), at minimal points everything is retrospectively necessary but nothing is retrospectively possible.

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Role of Logic in Cognitive Science

Paweł Balcerak

University of Rzeszów, Poland

e-mail: pbalcerak@gmail.com

Abstract:
In their work McCulloch and Pitts describe an idea of representing all of nervous activity in terms of propositional logic. This idea was quickly challenged. One of reasons for this challenge was rising believe that logic is unable to describe most of human cognitive processes. In this paper we will analyse premises of original McCulloch and Pitts proposition. Following that, we will ask about ability of symbolic (logical) systems to represent human cognition. We will finish by analysing relation between symbolic and subsymbolic computing, in hope of bridging the gap between the two.
Keywords: nonmonotonic logic; neural networks; human reasoning.

1. Introduction

The gap between symbolic and subsymbolic (neural network) modes of computation is a riddle for the philosophy of mind. Complex symbolic systems like those of grammar and logic are essential when we try to understand the general features and the peculiarities of natural language, reasoning and other cognitive domains. On the other hand, most of modern theories assume stance seeing that cognition resides in the brain and that neuronal activity forms its basis. Yet neuronal computation appears to be numerical, not symbolic; parallel, not serial; distributed over a gigantic number of different elements, not as highly localized as in symbolic systems. Moreover, the brain is an adaptive system that is very sensitive to the statistical character of experience. “Hard-edged” rule systems (classical logic) are not suitable to deal with this aspect of behavior. We will start with analyzing the roots of neural network approach, seen here as paradigmatic example of subsymbolic computation approach. It is widely accepted that this method started with the work by Warren S. McCulloch and Walter H. Pitts titled *A Logical Calculus of the Ideas Immanent in Nervous Activity* [16]. We will try to show connections between this approach and logical description of reasoning processes.

In the early days of cognitive science, logic was taken to play both a descriptive and a normative role in theories of intelligent behavior. Descriptively, human beings were taken to be fundamentally logical, or rational. Normatively, logic was taken to define rational behavior and thus to provide a starting point for the artificial reproduction of intelligence. Both positions were soon challenged. As it turns out however, logic continues to be at the forefront of conceptual tools in cognitive science. What is embodied by competitive to connectionist (neural network) AI approach. Rather than defeating the relevance of logic, the challenges posed by cognitive science have inspired logicians to enrich the repertoire of logical tools for analyzing reasoning processes and computation. We will examine the role of nonmonotonic logics in this endeavor. This kind of logic allows to overcome logics problem to deal with "soft-edged" rules, that neural networks excel at.

2. Logic and Neuroscience

Logic is a brand of science that deals with studying of the *correct reasoning*. Reasoning is a mental activity and as such is seen as at least closely related to the way the mind works. Classically, in logic the correct reasoning was synonymous with deductive reasoning and ordinary deductive reasoning takes place in natural language. That is why, to answer the question about the role of logic in science about cognition, we have to first ask about the relation between natural and formal language. As stated above, logic had two dimensions to its research, descriptive and normative theories of intelligent behavior. Those two dimensions find their explication in two kinds of answers to the question about natural-formal language relation. First view states that, at least some sentences of natural language have underlying logical form and these form are represented by formulas of formal language – this view is compatible with the descriptive dimension of logic. Since reasoning is an activity performed in language, logic provides deep structure of correct reasoning. This view is represented by philosophers such as Davidson [3]. The second view is that natural languages are ambiguous and vague and as such should be replaced by formal language lacking these features – this view is compatible with normative dimension of logic. According to a view like this, logically correct reasoning represents ideal sought after activity in natural language. In philosophy this approach can be found in works of W.V.O. Quine [19]. With the rise of cognitive science both of those roles were put into question. Instead of eliminating logic out of cognitive science it motivated logicians to expand tools in their repertoire.

Parallel to modern logic, a different type of science has begun its emergence since late 19^{th} century. One that examined physical basis, rather than abstract rules governing the work of human mind. It was called neuroscience and it seemed as if nothing connected the two activities. It began to change with the publication of *A Logical Calculus of the Ideas Immanent in Nervous Activity* at the end of first half of 20^{th} century. This paper is often cited as the starting point of research in artificial neural networks; for us it is the first moment in which research fields of logic and neuroscience meet. McCulloch and Pitts state in their paper that activity of any neuron may be represented as a proposition. We can assert that relations existing among nervous activities can be represented as relations between propositions. They notice two difficulties immanent in this approach, both problems rising from the physiological aspects of nervous activity. The first concerns the effects of previous excitations on future activations of nervous cells. The second notices that learning has to be a permanent change in neural structure. Nonetheless, they see this only as problematic in the case of asserting factual equivalency (or identity) between calculus of logical propositions and neural structures. Their statement is of much weaker kind; physiological aspects of neural systems do not affect the fact that relations of propositions corresponding to certain nervous activities are that of propositional logic.

Because of that they make certain assumptions about their calculus. These assumptions are aimed at simplifying of the behavior of real neurons.

(1) Activity of neurons is binary, they are either on or off.
(2) The threshold of neuron activation is independent of previous activations of a neuron.
(3) The only delay significant for nervous activity is the synaptic one.
(4) Inhibitory synapses absolutely prevent activation of neuron at certain moment.
(5) The structure of neural net does not change in time.

All of the above assumptions seam necessary to represent the neural activity in logical calculus. Additionally they arise as a result of the difference between formal and factual equivalency, authors distinguished. The actual neural activity would not comply to such rules, but the idea is – as stated before – that they talk about the abstract calculus of "mind".

The authors divide neurons into two categories. One that they name *peripheral afferents* - input neurons that do not receive signals from any other neuron in the net. Second consisting of all other neurons. Next step they take, consists of developing a logical apparatus necessary to define basic concepts of their calculus. As noted by Stephen C. Kleene [12] the approach and notation used by McCulloch and Pitts are obscure and hard to understand, that is why we will try to streamline it and present in a more approachable manner. Let us consider two problems presented by the authors: "(…) first, to find an effective method of obtaining a set of computable *S* constituting a solution of a given net [16, p. 103]."

In other words, an answer to the question: what does a given net compute (How to calculate behavior of the net)? This is called the *solution* of a net. We can define the solution of a net as a set of logical sentences of the form: neuron *i* is firing if and only if a given logical combination of the firing predicates of input neurons at previous times and some constant sentences including firing predicates of these same neurons at $t=0$ is true. These sentences are the solution for a net if they are all true for it.

The second problem is characterized as follows: "(…) to characterize a class of realizable *S* in effective fashion (ibid. 103)." The question here can be summarized as: can a certain net compute a given logical sentence (How to find a net that behaves in a specific way)? A sentence is *realizable* for a net if it is true for that net, or in other words when a net can compute it.

Following Stenning and van Lambalgen [21, pp. 218-219] we can define net, in modern fashion, as follow:

Definition 1 *Net is a graph on a set of computational units, connected with weighted links that can be either excitatory of inhibitory.*

Accordingly units can be defined:

Definition 2 *Computational unit (unit) is a function with the following behavior:*

- *Inputs are delivered through weighted links* $w_j \in [0, 1]$.
- *Links can be either excitatory* ($x_{1,\ldots,} x_n \in \mathbb{R}$) *or inhibitory* ($y_1,\ldots, y_n \in \mathbb{R}$).
- *If an inhibitory link is active* ($y_i \neq 0$)*, connected unit is shut off, and outputs 0.*
- *Otherwise, quantity* $\sum_{i=1}^{i=n} x_i w_i$ *is calculated; if it equals or exceeds threshold* (Θ) *unit is active and outputs 1; otherwise, unit rests and outputs 0.*

We can represent logical connectors in terms of units and connections. Conjunction can be represented by unit witch two excitatory inputs and threshold of 2; alternative can be represented by unit witch two excitatory inputs and threshold of 1; negation can be represented by unit witch one excitatory input and one inhibitory.

Authors propose a class of expressions representing solution of net, called *temporal propositional expressions* (TPE). TPEs have a single free variable, identified as discreet time.

Definition 3 *TEPs are defined by the following recursion:*

- *Predicate of one argument is a TPE.*
- *Logical disjunction, conjunction and negated conjunction (and not) of TPEs with the same free variable are by themselves TPE.*
- *Nothing else is a TPE.*

Theorems 2 and 3 of the discussed work give us a version of a rule of substitution for neural nets and a set of basic expressions from which those expressions can be constructed. Rule of substitution can be summarized as follows: *replacing peripheral afferent in a realizable net by a realizable net is in itself a realizable net.* By that definition all TPE are realizable. Set of basic realizable expressions follows then from definition of TPE and consist of nets representing operations of precession, disjunction, conjunction and negated conjunction. Respectively each net is represented below by figures 1a-d. Lines witch arrows at ends represent excitatory connections, lines witch circles at the ends represent inhibitory connections.

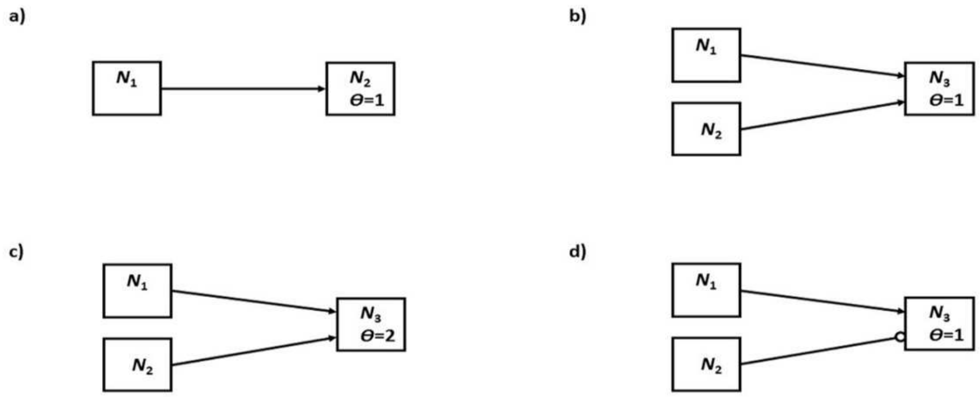

Figure 1. a) precession; b) disjunction; b) conjunction; c) negated conjunction. Version of nets presented in McCulloch and Pitts [16] adapted to presented definitions.

It can be described by following expressions:
a) $N_2(t) \equiv N_1(t\text{-}1)$
b) $N_3(t) \equiv N_1(t\text{-}1) \vee N_2(t\text{-}1)$
c) $N_3(t) \equiv N_1(t\text{-}1) \wedge N_2(t\text{-}1)$
d) $N_3(t) \equiv N_1(t\text{-}1) \wedge \sim N_2(t\text{-}2)$

The rule of substitution, following from mentioned theorems gives us a simple procedure of constructing neural nets. The authors propose to consider an example of heat sensation evoked by a short time cooling [16, pp. 106-107]. If a cold object makes contact with the skin and is instantaneously removed, the sensation of heat will occur; if the same object will not be removed, the sensation of cold occurs without the preliminary heat sensation. This happens for cold receptors but not for heat receptors. We assume there are different receptors responsible for heat and cold detection, but the same neuron is responsible for heat sensation in both cases. Because of that, the synaptic delay for the sensation of cold must be greater by one then for the sensation of heat. We can reproduce this effect using the described method by transforming the above mentioned expressions using the rule of substitution. We receive:
e) $N_3(t) \equiv N_1(t\text{-}1) \vee [N_2(t\text{-}3) \wedge \sim N_2(t\text{-}2)]$
$N_4(t) \equiv N_2(t\text{-}2) \wedge N_2(t\text{-}1)$

We can notice this net has 2 solutions, one for heat and one for cold respectively. Figure in which both of those expressions are realizable can be constructed from figures 1a-d in the following manner.

Beginning in the standard logical manner, we first consider the function enclosed in most brackets. We receive a net of form 1a representing expression:

$$N_a(t) \equiv N_2(t\text{-}1) \tag{1}$$

Proceeding outwards, we introduce two nets, both starting from nodes N_a and N_2. One of form 1c ending in N_4. We receive:

$$N_4(\mathrm{t}) \equiv N_a(t\text{-}1) \wedge N_2(t\text{-}1) \tag{2}$$

We must advance time variable for previous expression where we substitute it in this formula. Which is equivalent to:

$$N_4(t) \equiv N_2(t\text{-}2) \wedge N_2(t\text{-}1) \quad (3)$$

Second of form 1d ending in N_b. Giving us:

$$N_b(t) \equiv N_a(t\text{-}1) \wedge {\sim}N_2(t\text{-}1) \quad (4)$$

Substituting N_a for its equivalent in proper time interval we receive:

$$N_b(t) \equiv N_2(t\text{-}2) \wedge {\sim}N_2(t\text{-}1) \quad (5)$$

Finally we run net of form 1b starting in N_1 and N_b to neuron N_3. So that:

$$N_3(t) \equiv N_1(t\text{-}1) \vee N_b(t\text{-}1) \quad (6)$$

Again, due to substituting N_b for equivalent formula, (6) can be expressed as:

$$N_3(t) \equiv N_1(t\text{-}1) \vee [N_2(t\text{-}3) \wedge {\sim}N_2(t\text{-}2)] \quad (7)$$

The whole net can be represented by figure 2.

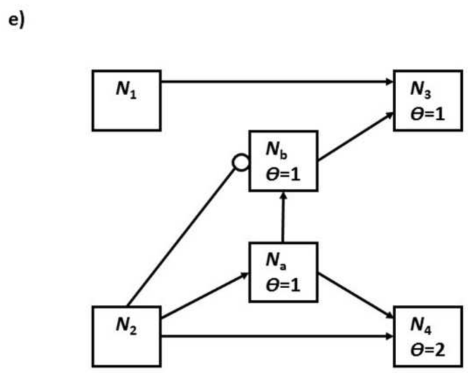

Figure 2. Net realizing expressions e). Modified from McCulloch and Pitts [16], to adapt to presented definitions.

That way we can create nets realizing underlying logical functions. We can clearly see that McCulloch saw propositional logic as an underlying structure of human mind. He writes:

> To psychology, however defined, specification of net would contribute all that could be achieved in that field – even if analysis were pushed to ultimate psychic unit or "psychon", for psychon can be no less than the activity of a single neuron. Since that activity is inherently propositional, all psychic events have an intentional, or "semantic" character. The "all-or-none" law of these activities, and the conformity of their relations to those of the logic of propositions, insure that relations of psychons are those of two-valued logic of propositions [16, pp. 113-114].

This sentence presents author's intentions of proving logical character of human mind activity. The nervous system is described as based on mechanics equivalent to propositional logic. Unfortunately, it highlights weak points of both logical approach and neural nets of McCulloch-Pitts type. This effort to "marry" logic and neuroscience marks the first and last attempt to do so by way of classical propositional logic. It may be because it highlighted certain weaknesses of logical approach – weaknesses we will analyze in the following paragraph.

3. Logic and Human Cognition

The above described neural networks meet with plenty of critique. Some of it is coming from biological background. For example, it was quickly noticed that the assumption about neurons always being in one of two possible states is biologically inadequate. In the context of discussion presented in this paper, what is more important is the fact that some developments in research of human cognition put descriptive dimension of logic under doubt. It remained a possibility that logic described a normative system of what certain types of reasoning should be, but it no longer could be perceived as a representation of natural cognitive processes.

If we accept descriptive dimension of logic, then at some level human reasoning should be based upon a set of simple logical procedures. However, humans tend to do surprisingly poorly when faced with tasks of performing simple logical procedures. This phenomenon was noticed and described by Wason in, named after him, *Wason Selection Task* [23], [24]. The task puts a subject in choice situation guided by a simple rule. The choice is made between cards. Each card has on it either a number or a letter. Cards, on a side visible to subject, read D, K, 7 and 3. The subject is then familiarized with singular rule of the task; "Every card which has D on one side must have a 3 on other". After that the question is posed; "Which if any, of the cards must be turned over to judge if the rule is true". From the classical logic standpoint the "if" in the rule should be read as material conditional, making the rule D → 3. Hence, using modus ponens (MP), we may deduce that D has to be turned to check if there is 3 on back side. Likewise, using modus Tollens (MT), we deduce 7 has to be turned over to ascertain if there is no D on the reverse. Making, assumed, correct answer D and 7. The most popular answer given is however, D and 3. In fact D is almost always given as one of the answers. Conversely, 7 is rarely seen as necessary to turn over. Some researchers, including Wason, see that as an evidence that humans are poor at even simple tasks. If we would accept Wason's interpretation of "D → 3" rule, we have to accept that people are bad at using MT, so tasks requiring it as reasoning schemata lead to fallacious reasoning.

Interesting development appeared out of certain rephrasing of Wason task [8], [11]. The original selection task took place in abstract domain of letters and numbers. Rephrasing the problem in a domain familiar to subjects changed outcome drastically. In the mentioned rephrasing, numbers and letters were replaced by ages and kinds of drinks. When the task is to confirm a rule "if person drinking beer, then that person is 19 or older", subjects performed nearly perfectly. Noticing the fact that rephrasing Wason's task in a familiar domain brings error rate down contradicts formal-logical model of reasoning.

The fact that context has an effect on the ability of subjects to deduce a correct answer may be explained by the theory of two competing systems of reasoning. It can be reasonably doubted that experiments like *Wason selection task* test what authors actually believed they did. Question can be posed: what does actually count as reasoning in natural environment? Proposing dual process theory of reasoning can explain the described situation. Here we assume reasoning consists of two systems supplementing each other. Describing *system 1* Evans writes:

> System 1 is (…) not a single system but a set of subsystems that operate with some autonomy. System 1 includes instinctive behaviors that would include any input modules of the kind proposed by Fodor.(…) The System 1 processes that are most often described, however, are those that are formed by associative learning of the kind produced by neural networks.(…) System 1 processes are rapid, parallel, and automatic in nature; only their final product is posted in consciousness [5, p. 454].

By contrast, *system 2* is slow, sequential and symbolic in nature. Logical reasoning belongs in system 2, because of that tasks performed by system 1 do not conform to rules of logic. This is also a reason why neural networks cannot be logical machines – system 1 is equivalent to a subsymbolic computing system.

We then have two approaches to reasoning. Let us call the first algorithmic: it states MP-MT asymmetry in Wason selection task is an effect of MT being harder to implement on algorithmic level. A sample of this approach can be found in Oaksford and Chater *The Probabilistic Mind: Prospects for Bayesian Cognitive Science* [17]. That is why reasoners trying to reason deductively have problems with finding the correct solution. The second, called non-logical reasoning, argues that subjects do not attempt to deductively find solutions to posed questions. That way MP-MT asymmetry is not a matter of competency gap but rather "inadequacy" of utilized competences.

Authors Stenning and van Lambalgen [21] propose a different analysis of context effect on task results. They attack Wason's assumption that "if" in the rule has to be interpreted as a material conditional, which puts doubt on the assertion that there is only one correct answer. They propose to distinguish between the two forms of conditionals: one descriptive; other deontic. That may explain why two statements (Wason task and Wason task rephrased in familiar context), of supposedly the same logical form can lead to radically different outcomes. The task when rule is seen descriptively, is viewed by subject, as concerning determining if the rule is true or false for the given cards. With deontic interpretation of conditional truth of the rule is not an issue, only whether the rule is being followed or not. They notice that the original task may be interpreted as containing descriptive rule, increasing the cognitive burden on subjects. However, in the context of this paper, the more important aspect is the observation of the processing side nonmonotonic logic provides adequate model for analysis of subjects' reasoning. Presenting human reasoning in terms of nonmonotonic logic explains why reasoning in a system which could not be explained in terms of logic. More precisely it is cold but not in classical logic. This system can still be represented by a set of reasoning rules, just not build upon deductive inferences. In this view, deontic interpretation of the rule can be associated with classical logical conditional, when descriptive interpretation entails a different kind of conditional, nonmonotonic, that should be read "typically this X entails Y".

To answer what differentiates classical logic from the nonmonotonic one, let's consider the following property of deductive logic, one that holds for relation of classical consequence "$\vDash$":

Monotony: if $A \vDash B$ then $A \cup C \vDash B$.

Monotony states that if B is a logical consequence of A, then it is also a consequence of any set containing A as its subset. In other words, adding a new premise to inference cannot pre-empt earlier conclusions. Monotony follows straight from nature of logical consequence relation, $A\vDash B$ holds when B is true on every interpretation on with every sentence in A are true. Clearly, every day inferences do not conform to this requirement. Actually, not abiding to it is a defining property of so called defeasible reasoning, the kind of nonmonotonic inference that supposedly describes how every day reasoning works. Literature is rich in analyses of reasons why deductive reasoning is inadequate in describing the so called everyday inferences [4], [18].

There are many examples of nonmonotonic logics, but for our purpose semantic approach of Shoham [20] will be used. This theory is often referred to as *preferential logic*, it is a simple and elegant approach. Additionally it can be used to explain the MP-MT asymmetry and perceived system 1 - system 2 dichotomy.

Definition 4 *$L_\angle$ is a nonmonotonic preferential logic generated from L and $\angle$ when following demands are met:*

- *In a standard logic L that satisfy following demand: for all A, B and C in L, if $A\vDash B$, then also $A\wedge C \vDash B$.*
- *A strict partial order $\angle$ on the model of L is defined: $M_1\angle M_2$, meaning that M_2 is preferred over M_1.*
- *Preferred model is one that: Model M preferentially satisfies A ($M\vDash_\angle A$); $M\vDash A$ and there is no other model M' such that $M\angle M'$. We call M preferred model of A.*

We can define a preferential consequence relation for that logic in the following fashion:

Definition 5 *Preferential consequence: A is a preferential consequence of B ($A\rightarrow_\angle B$) for any M, if $M\vDash_\angle A$, then $M\vDash B$.*

In other words $A \rightarrow_{\angle} B$ if all preferred models of A are models of B. This relation is nonmonotonic because it is possible that A and C have preferred models that are not preferred models of A alone. So with addition of C it may be that that B no longer holds in all preferred models of $A\&C$.

Now we can notice that preferential consequence relation easily explains MP-MT asymmetry. It refers to preferred models of A, but also to all models of B. Because of it, this consequence relation does not contrapose. For the relation to be contrapositive it would be required that all preferred models of not-B be models of not-A. It is quite possible there exist not-preferred models of A wich are also preferred models of not-B. Thus, the definition is not satisfied for not-$B \rightarrow_{\angle}$ not-A, and MP-MT asymmetry is explained.

4. Symbolic vs. Subsymbolic Paradigms

Classical view of human cognition is one analogous to symbolic computation in digital computers [23]. On this account information is represented as a string of symbols in memory of a computing unit or on a piece of paper. On the other hand connectionist claim that information storage have a non-symbolic character, information is stored in weights of connections between units of neural net. Connectionists perceive mental processes as dynamic and distributes evolution of activity in neural net. Each unit of this net activates depending on strength of connections and activity of neighboring units.

In late 20th century a heated debate ensued between proponents of symbolic and connectionist (subsymbolic) approach to cognitive science. One of most vocal opponents of connectionism were J. Fodor and Z. Pylyshyn [6]. They argued that no connectionist model of mind can have compositional semantics. That is the case because, as they argued, mental representations require systematicity and no neural network can exhibit this feature; therefore modeling of cognition have to be symbolic not connectionist. Systematicity is understood as a feature of representation that makes meaning of representation to correspond systematically to its structure. That means if we are able to represent expression “Peter killed Paul”, we must be able to represent expression “Paul killed Peter”. Putting details of this debate aside, prevailing view was that symbolic and subsymbolic approach are different and incompatible.

Concurrently, radical connectionists claimed inadequacy of symbolic processing as a model of mind. We discussed this in part 3 of this paper. To reiterate, they claimed that symbolic computing poorly explains holistic representation of data, spontaneous generalization, effect of context, and many other aspects of human cognition captured by their models. This failure to match the flexibility and efficiency of human cognition is in their eyes a symptom of the need for a new paradigm in cognitive science. This approach can be called *radical connectionism,* and it agenda can be described as eliminating symbolic processing as inadequate in cognitive science.

However, many connectionists do not view their paradigm as opposition to symbolic computation. So called implementation connectionists present an image in with mind is a neural net, but also a symbolic process on higher level of abstraction. In that view role of connectionist researcher is to find how a machine required to perform symbolic processes can be forged from neural network resources. Even more interestingly since 1990’s, models combining subsymbolic and symbolic paradigms appeared [1], [7], [25]. Unfortunately hybrid approach to problem fails address question about underlying difference between symbolic and distributed representation. Because of that it is proposed to inquire about possible equivalency between symbolic and subsymbolic models of computation.

The idea is that connection can come again from the side of logic, similarly to original McCulloch and Pitts proposition. Instead of classical logic we would turn to nonmonotonic one. This way we avoid problems with inadequacy of logical description to data collected during research on human cognition. The close relation between symbolic computation and logic is well known [10]. With neural nets it have to be shown that every logical model of a system is isomorphic to a member of distributed, or subsymbolic, subset. In fact, it is trivial to show that some nonmonotonic reasoning may be represented by neural networks. An example of neural net generatinc a nonmonotonic inference was shown as early as 1991 [2]. They propose to consider network consisting of four neurons $x_1, \ldots,$ x4. They identify sets of active neurons with schemata. There are three schemata α, β, γ. Corresponding to following sets of active neurons: $\alpha = x_1, x_2$; $\beta = x_2, x_3$; $\gamma = x_4$. There are two excitatory connections, one between x_1 and x_2, other between x_2 and x_3. Third connection between x_4 and x_3 is inhibitory connection. Assuming that inhibitory connection is stronger than excitatory one between x_2 and x_3. Following situation is possible: giving α as input, the network will activate β ($\alpha \vDash \beta$); extending inputs to α and γ, effects in withdrawal of β ($\alpha \wedge \gamma \nvDash \beta$). That situation directly defy monotonicity, since including new premises (inputs) reduce set of conclusions.

However, this is just one specific case when neural network exhibits behavior equivalent to some nonmonotonic theory. Can we have an equivalency theorem? Theorem of that kind would show that for every nonmonotonic theory there exist a neural network able to compute that theory. Fortunately theorems of that kind has been proposed by logicians over the last few decades. Holldobren and Kalinke [9] gives a theorem of that kind. They show that for every logical program there exists a three layer feed forward network which computes it. Other example is presented by Leitgeb [13], [14], [15]. His proposition is especially interesting in context of this debate. He propose a way to represent propositional letters as a set of nodes in neural networks. At the same time Leitgeb shows that any dynamic system performing calculations over distributed representation can be interpreted as symbolic system performing nonmonotonic inferences. What can be interpreted as functional equivalence of reasoning representation between symbolic and subsymbolic processes.

5. Conclusions

The methodological position pursued in this article was one which looks for unification. In the case under discussions the point was to assume that symbols and symbol processing are a macro-level description of what is considered a connectionist system at the micro level. Hence, the idea is that the symbolic and the subsymbolic mode of computation can be integrated within a unified theory of cognition. We demonstrated that logical approach can be applied to model and describe processes of human reasoning, previously regarded as evading symbolic representation. Which leads to believe that, at least functionally, neural network activity is equivalent to nonmonotonic inferences.

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On Logics of Transitive Verbs With and Without Intersective Adjectives

Selçuk Topal

Bitlis Eren University,
Turkey

*e-mail:*s.topal@beu.edu.tr

Abstract:
The purpose of this paper is to contribute to the natural logic program which invents logics in natural language. This study presents two logics: a logical system called $\mathsf{R}(\forall,\exists)$ containing transitive verbs and a more expressive logical system $\mathsf{R}(\forall,\exists,IA)$ containing both transitive verbs and intersective adjectives. The paper offers three different set-theoretic semantics which are equivalent for the logics.
Keywords: Logic of natural languages; adjectives; transitive relations; transitive verbs; relational syllogistics.

1. Introduction

Relational syllogistic theories have been taking place in wide applications of different areas such as in natural language theory and generalized quantifiers [5], [1], [7], [9], [8], [22], in algebraic structures [2], [3], [16], [20], in formal logic [4], [11], [12], [15], [14], [17]. The Aristotelian syllogistic did not touch on the validity of sentences containing transitive verbs. De Morgan presented traditional syllogism within relational facts [6]. De Morgan did not mention syllogisms with binary relations with the intention of transitive verbs. Hartmann and Moss extended syllogism with binary relations with the aim of using transitive verbs [17]. Moss presented a logical study using of intersective adjectives in basic syllogistic [12]. Nikolay and Dimiter presented a system of relational syllogistic, based on classical propositional logic and Stone theory [10].

This paper considers the so-called informative verbs. In its atomic propositions "QS + $verb$ + QS" and "QS + $verb$ + QP_1 + to + QP_2" where $Q \in \{some, all\}$. These verbs designate actions which can be observed and are not depended on their utterances ('to run', 'to take', etc.). However, there are also the so-called performative verbs. They are carried out only by means of uttering them aloud ('to love', 'to hate', etc.). The syllogistic for performative propositions is first introduced in [18]. In this system, there are examined concepts which have no denotations at all verbs such 'love', 'hate', etc. For these concepts, therefore, we can not define an inclusion relation and we need a novel formal system. Some applications of that new syllogistic are proposed in [18], [19].

The current author of this paper presented algebraic semantics (bounded meet semi-lattice) of binary and ternary relational logics by using congruence theory [21]. This paper offers some different semantics for $\mathsf{R}(\forall,\exists)$ and $\mathsf{R}(\forall,\exists,IA)$.

1.1.Some Explanations on Inference Patterns and Languages of the Logics

In this paper, we study three different equivalent set-theoretic models for inference patterns of sentences in natural language related to intersective adjective phrases in binary relational (transitive verbs) syllogistics. In this sense, there are two logics $\mathsf{R}(\forall,\exists)$ and $\mathsf{R}(\forall,\exists,IA)$ which is a follow-up the work of Moss [20]. Sentences of the language of $\mathsf{R}(\forall,\exists)$ consist of two quantifiers "*for all*" ($\forall$) and "*exists*" ($\exists$), and plural nouns and also transitive verbs, but $\mathsf{R}(\forall,\exists,IA)$'s also include *intersective adjectives*. Our approach to sentences with or without intersective adjectives falls in model-theoretic semantics. The interpretation of a phrase such as *red cars* would be the intersection of the interpretation of "red things" and a set of "car individuals".

English sentences such as "*all students love some cleaver teachers*" are ambiguous. We use these kinds of sentences in meaning of "there is at least one cleaver teacher who all students love". In this regard, the sentences reflect binary relational perspective directly in our logics. On the other hand, we are not interested in sentence forms of Aristotle's syllogistic which consists of $Det + A$ are (are not) + B where Det is All or $Some$ or No, and also A and B are plural nouns but $Det + A$ + *transitive verb* + $Det + B$.

Universal quantifiers entail existential quantifiers in our logics because the interpretation of nouns does not allow to be empty set as is in Corcoran's syllogistic system [5]. Some examples of the inference patterns in our languages as follows:

(i) Some students love all teachers
-- (I1)
Therefore, some students love some teachers

(i) Some cleaver students see all teachers
(ii) Some instructive teachers see some janitors
--- (I2)
Therefore, some students see some instructive teachers

Inferences in Aristotle's syllogistic let sentences to obtain nouns in their conclusions from different the ones in their premises. Although the plural noun *educators* is not be contained by the premise (i), it does by the conclusion as can be seen in (I3).

(i) Some students see all teachers
(ii) All teachers are educators
--- (I3)
Therefore, some students see some educators

Turning to binary syllogistic $\mathsf{R}(\forall,\exists)$ without Aristotle's, one must make inferences with sentences having the same relations, the same nouns and the same orders both in premises and in conclusions as in example (I1). Under the circumstances, the changes must be situated in quantifiers in derivations of the syllogistics but no changes for nouns and relations. On the one hand, the unchangeability of nouns and relations force the structure $\mathsf{R}(\forall,\exists)$ to have equivalence classes (see remarks 2.11 and 3.10). Concerning with binary syllogistic $\mathsf{R}(\forall,\exists,\mathsf{IA})$ without Aristotle's, the plural adjectival noun *instructive* *teachers* is not be contained by the premise (i) but it is contained by the conclusion as can be seen in (I2). This indicates that if there is an intersective adjectival noun in premises, we may have it in conclusion to restrict inferences by intersective adjectival nouns. This situation induces to force using of equivalence classes within the structure $\mathsf{R}(\forall,\exists,\mathsf{IA})$. In other words, if there is no intersective adjectival noun in premises, we can not make an inference containing intersective adjectival nouns.

Finally, notice that the set of nouns and relations have countable sizes and all models are finite throughout the paper. Languages of the logics in this paper are not closed under boolean operations and do not have recursion.

2. The Logic of $\mathsf{R}(\forall,\exists)$

Our syntax starts with a collection P of *unary atoms* (for nouns) and another collection, R of *binary atoms* (for transitive verbs). A transitive verb takes a subject and a direct object - shall be interpreted as a binary relation on the universe M.

$$\frac{\forall(p,\forall(r,q))}{\exists(p,\forall(r,q))}\ (1) \qquad \frac{\forall(p,\forall(r,q))}{\forall(p,\exists(r,q))}\ (2) \qquad \frac{\exists(p,\forall(r,q))}{\exists(p,\exists(r,q))}\ (3) \qquad \frac{\forall(p,\exists(r,q))}{\exists(p,\exists(r,q))}\ (4)$$

Fig. 1. Rules for $\mathsf{R}(\forall,\exists)$

Observation 2.1. *An unsound inference:*

$$\frac{\forall(p,\forall(r,q) \quad \exists(p,\forall(r,q))}{\forall(p,\exists(r,q)}$$

To see the rule is not sound, we construct a counter-model. Suppose that $[[p]]=\{p_1,p_2\}$ and $[[q]]=\{q_1,q_2,q_3\}$ and also $[[r]]=\{(p_1,q_1),(p_1,q_2),(p_1,q_3),(p_2,q_1),(p_2,q_2)\}$. Whereas the premises are true in the model, the conclusion $\forall(p,\forall(r,q))$ is false.

Syntax	Reading of Syntax	Natural Example
$\forall(x,r(\forall,y))$	**All x r all y**	**All students love all teachers**
$\forall(x,r(\exists,y))$	**All x r some y**	**All students love some teachers**
$\exists(x,r(\forall,y))$	**Some x r all y**	**Some students see all teachers**
$\exists(x,r(\exists,y))$	**Some x r some y**	**Some students see some teachers**

Table 1. Syntax, their natural readings, natural examples

Lemma 2.2. *Let* Γ *be a set of sentences in* $\mathsf{R}(\forall,\exists)$. *The followings hold:*

1. $\Gamma \vdash \forall(p,\forall(r,q))$ *if and only if* (iff) $\forall(p,\forall(r,q))\in\Gamma$.
2. If $\Gamma \nvdash \forall(p,\forall(r,q))$ and $\Gamma \vdash \exists(p,\forall(r,q))$, then $\exists(p,\forall(r,q))\in\Gamma$.

3. If $\Gamma \not\vdash \forall(p,\forall(r,q))$ and $\Gamma \vdash \forall(p,\exists(r,q))$, then $\forall(p,\exists(r,q)) \in \Gamma$.

4. If $\Gamma \not\vdash \forall(p,\forall(r,q))$ and $\Gamma \not\vdash \forall(p,\exists(r,q))$ and $\Gamma \not\vdash \exists(p,\forall(r,q))$ and $\Gamma \vdash \exists(p,\exists(r,q))$, then $\exists(p,\exists(r,q)) \in \Gamma$.

2.1.Model Construction

Here, we give some definitions and examples to clarify the paper.

Definition 2.3. *P is a set of noun variables, $\forall$ and $\exists$ are quantifiers in language of the logic. $\vec{P}$ is a set which consists of elements which accepted quantifiers in the language as subscript of nouns.*

Example 2.4. *If $P = \{x, y\}$, then $\vec{P} = \{x_\forall, y_\forall, x_\exists, y_\exists\}$.*

Definition 2.5. *Let Γ be a set of sentences. P_Γ is the set of nouns occurring in Γ. R_Γ is the set of binary terms in Γ. $\vec{P}_\Gamma$ is the set of elements of P_Γ with their quantifiers.*

Example 2.6. $\Gamma = \{\forall(x,\exists(r_0, y)), \exists(x,\exists(r_1, y)), \forall(z,\forall(r_1, h))\}$.

$R_\Gamma = \{r_0, r_1\}$, $P_\Gamma = \{x, z, y, h\}$, $\vec{P}_\Gamma = \{x_\forall, z_\forall, x_\exists, y_\exists, h_\forall\}$

Definition 2.7. *We define an translation from $\vec{P}$ to $\mathsf{P}(\vec{P})$ as the following:*

$$[\ ] : \vec{P} \mapsto \mathsf{P}(\vec{P})$$
$$x_\forall \mapsto \{x_\forall, x_\exists\}$$
$$x_\exists \mapsto \{x_\exists\}$$

Definition 2.8. *We define two sets* $[\vec{P}_\Gamma] = \{[i] : \text{for i in } \vec{P}_\Gamma\}$ *and* $M^+ \subseteq \vec{P} \times \vec{P} \times R$.

Definition 2.9. *Let Γ be a set of sentences and $\Gamma_{Vec} \subseteq [\vec{P}_\Gamma] \times [\vec{P}_\Gamma] \times R_\Gamma$. We define a translation from Γ to Γ_{Vec}.*

$$\Upsilon_V : \Gamma \mapsto \Gamma_{Vec}$$
$$\alpha(p, \beta(r,q)) \mapsto ([p_\alpha], [q_\beta], r)$$

Please notice that the translation is an one to one correspondence.

Remark 2.10. *Note that $\Gamma_{Vec} \subseteq M^+$.*

Definition 2.11. *Two elements $([k_\alpha],[l_\beta], r_0)$ and $([p_\gamma],[q_\theta], r_1)$ of Γ_{Vec} are in the same equivalence class, if $k = ax$ or $k = x$ and $p = ax$ or $p = x$ and $l = by$ or $l = y$ and $q = cz$ or $q = z$ and $r_0 = r_1$ where x, y, z are basic nouns.*

Remark 2.12. *If two elements in M^+ are in the same equivalence class, we will denote two elements* *that first two elements are represented by the same letters and last ones are the same. For instance, $([p_\alpha],[q_\beta],r)$ and $([p_\gamma],[q_\theta],r)$ are in the same equivalence class because first objects of two elements are denoted by p, second ones are q and last ones are r.*

Definition 2.13. *A down-set of element $([k_\alpha],[l_\beta],r_0)$ of M^+ is a set $d^R_{\Downarrow}[([k_\alpha],[l_\beta],r_0)] = \{([p_\alpha],[m_\beta],r_1):[p_\alpha]\subseteq[k_\alpha] \text{ and } [m_\beta]\subseteq[l_\beta] \text{ and } r_0 = r_1\}$. We also define $d^R_{\Downarrow}[M^+] = \{d^R_{\Downarrow}[i]: i \in M^+\}$, shortly, $\overset{\triangleright}{M}{}^{+}_{\Gamma}$.*

Definition 2.14. $[p_\alpha]\subseteq[k_\alpha] \text{ and } [m_\beta]\subseteq[l_\beta] \text{ and } r_0 = r_1$ *iff* $([p_\alpha],[m_\beta],r_1)\subseteq([k_\alpha],[l_\beta],r_0)$.

Theorem 2.15. $\Gamma|-\alpha(p,\beta(r,q))$ *iff* $([p_\alpha],[q_\beta],r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$*, in other words,*

$\mathsf{M}_R = (M_R,[[\,]]) :\Leftrightarrow \mathsf{M}_R = (\overset{\triangleright}{M}{}^{+}_{\Gamma},\in)$.

Proof 2.15. We will prove the theorem on complexity of sentences of Γ and elements of $\overset{\triangleright}{M}{}^{+}_{\Gamma}$.

$(\Rightarrow)$:

(i) Suppose that $\Gamma|-\forall(p,\forall(r,q))$. It is clear by Lemma 2.2.

(ii) Suppose that $\Gamma|-\exists(p,\forall(r,q))$ and $\Gamma|\!\not-\forall(p,\forall(r,q))$. $\exists(p,\forall(r,q))$ must be in Γ by Lemma 2.2. So, $(\{p_\exists\},\{q_\forall,q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$.

(iii) Suppose that $\Gamma|-\forall(p,\exists(r,q))$ and $\Gamma|\!\not-\forall(p,\forall(r,q))$. $\forall(p,\exists(r,q))$ must be in Γ by Lemma 2.2. So, $(\{p_\forall,p_\exists\},\{q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$.

(iv) Suppose that $\Gamma|\!\not-\forall(p,\forall(r,q))$ and $\Gamma|\!\not-\forall(p,\exists(r,q))$ and $\Gamma|\!\not-\exists(p,\forall(r,q))$ and $\Gamma|-\exists(p,\exists(r,q))$, then $\exists(p,\exists(r,q))\in\Gamma$ by Lemma 2.2. Therefore, $(\{p_\exists\},\{q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$.

$(\Leftarrow)$:

(i1) Suppose that $(\{p_\forall,p_\exists\},\{q_\forall,q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$. It is clear by Lemma 2.2.

(i2)Suppose that $(\{p_\exists\},\{q_\forall,q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$ and $([p_\exists],[q_\forall],r)\notin\Gamma_{Vec}$. Then, $([p_\forall],[q_\forall],r)$ must be in Γ_{Vec} so that $(\{p_\exists\},\{q_\forall,q_\exists\},r)\in\overset{\triangleright}{M}{}^{+}_{\Gamma}$ by the model construction. $\Gamma|-\forall(p,\forall(r,q))$ by (i1). Finally, $\forall(p,\forall(r,q))|-\exists(p,\forall(r,q))$ by rule (1) in Figure 1.

Other proofs are routine.

Theorem 2.16. $\Gamma|-\varphi$ *iff there exists at least one* ψ *such that* $[\varphi]\subseteq[\psi]$ *in* $\overset{\triangleright}{M}{}^{+}_{\Gamma}$.

Proof 2.16. We saw that there is at least one upper set of φ to derive it from Γ or a sentence ψ due to the definitions Υ_V and down-sets in the sufficient condition of Theorem 2.15.

3. The Logic of R(∀,∃,IA)

Syntax: Our syntax begins with basic nouns $x, y, z...$ by adding intersective adjectives $a, b, c....$ We define the set of nouns, and denote nouns by letters like $n, p,$ and q, by saying that the basic nouns are nouns, and if x is a noun and a is an intersective adjective, then ax is a noun. We call these nouns of the form ax complex nouns. We do not allow productive predictions which allow to be used more than one adjective in a complex noun such as $ab : x$ where a and b are adjectives and x is a basic noun. One collection P of *unary atoms* (for nouns) and another collection, R of *binary atoms* (for transitive verbs). As is in R(∀,∃), verbs will be interpreted as binary relations on the universe M.

Syntax	Reading of Syntax	Natural Example
$\forall(x, r(\forall, y))$	**All x r all y**	**All students love all teachers**
$\forall(ax, r(\forall, by))$	**All ax r all by**	**All cleaver students love all instructive teachers**
$\forall(x, r(\forall, by))$	**All x r all by**	**All students love all instructive teachers**
$\forall(ax, r(\forall, y))$	**All ax r all y**	**All cleaver students love all teachers**
$\forall(x, r(\exists, y))$	**All x r some y**	**All students love some teachers**
$\forall(ax, r(\exists, by))$	**All ax r some by**	**All cleaver students love some instructive teachers**
$\forall(x, r(\exists, by))$	**All x r some by**	**All students love some instructive teachers**
$\forall(ax, r(\exists, y))$	**All ax r some y**	**All students love some teachers**
$\exists(x, r(\forall, y))$	**Some x r all y**	**All students see all teachers**
$\exists(ax, r(\forall, by))$	**Some ax r all by**	**All cleaver students see all instructive teachers**
$\exists(ax, r(\forall, y))$	**Some ax r all y**	**All cleaver students see all teachers**
$\exists(x, r(\forall, by))$	**Some x r all by**	**All students see all instructive teachers**
$\exists(x, r(\exists, y))$	**Some x r some y**	**All students see all teachers**
$\exists(ax, r(\exists, by))$	**Some ax r some by**	**All cleaver students see all instructive teachers**
$\exists(ax, r(\exists, y))$	**Some ax r some y**	**All cleaver students see all teachers**
$\exists(x, r(\exists, by))$	**Some x r some by**	**All students see all instructive teachers**

Table 2. Syntax, their natural readings, natural examples

Semantics: A model M is a set M, together with interpretation functions

$$[[\,]] : P \to \mathcal{P}(M)$$
$$[[\,]] : R \to \mathcal{P}(M \times M)$$

For each unary atom $p \in P$, $[[p]] \subseteq M$, and for each binary atom r, $[[r]] \subseteq M \times M_R$. We interpret set terms by subsets of M in the following way:

$$[[\forall(r,q)]] = \{x \in M : \text{for all}\, v \in [[q]], (x,v) \in [[r]]\}$$
$$[[\exists(r,q)]] = \{x \in M : \text{some}\, v \in [[q]], (x,v) \in [[r]]\}$$

Here is how set terms are read:

$\forall(r, by)$: those who r all by
$\forall(r, y)$: those who r all y
$\exists(r, by)$: those who r some by

$\exists(r, y)$: those who r some y

Finally, we have the definition of truth in a model:

$M_R \models \forall(p, r(\forall, y))$ iff $[[p]] \subseteq [[r(\forall, y)]]$

$M_R \models \forall(p, r(\exists, y))$ iff $[[p]] \subseteq [[r(\exists, y)]]$

$M_R \models \exists(p, r(\exists, y)$ iff $[[p]] \cap [[r(\exists, y)]] \neq \varnothing$

$M_R \models \exists(p, r(\forall, y))$ iff $[[p]] \cap [[r(\forall, y)]] \neq \varnothing$

$$\frac{\forall(p,\forall(r,q))}{\exists(p,\forall(r,q))}\ (1) \qquad \frac{\forall(p,\forall(r,q))}{\forall(p,\exists(r,q))}\ (2) \qquad \frac{\exists(p,\forall(r,q))}{\exists(p,\exists(r,q))}\ (3) \qquad \frac{\forall(p,\exists(r,q))}{\exists(p,\exists(r,q))}\ (4)$$

$$\frac{\exists(ax,\beta(r,q))}{\exists(x,\beta(r,q))}\ (5) \qquad \frac{\beta(p,\exists(r,ay))}{\beta(p,\exists(r,y))}\ (6)$$

$$\frac{\forall(x,\beta(r,q)) \quad \exists(ax,\alpha(r,h))}{\forall(ax,\beta(r,q))}\ (7) \qquad \frac{\forall(x,\beta(r,q)) \quad \alpha(h,\exists(r,ax))}{\forall(ax,\beta(r,q))}\ (8)$$

$$\frac{\beta(p,\forall(r,y)) \quad \alpha(h,\exists(r,ay))}{\beta(p,\forall(r,ay))}\ (9) \qquad \frac{\beta(p,\forall(r,y)) \quad \exists(ay,\alpha(r,h))}{\beta(p,\forall(r,ay))}\ (10)$$

Fig. 2. Rules for $\mathsf{R}(\forall,\exists,\mathsf{IA})$: p,h,q nouns, x,y,z basic nouns, $\beta,\alpha \in \{\forall,\exists\}$

$$\begin{tikzcd}
 & \forall(p,\forall(r,q)) \arrow[dl] \arrow[dr] & \\
\exists(p,\forall(r,q)) \arrow[dr] & & \exists(p,\forall(r,q)) \arrow[dl] \\
 & \exists(p,\exists(r,q)) &
\end{tikzcd}$$

Fig. 3. Derivation diagram for rules (1), (2), (3), (4), (5). p and q are basic nouns or complex nouns.

Figure 2 indicates rule set of the logic. The rules (6) – (11) are abbreviated form of too many rules. For example, $\frac{\exists(ax,\forall(r,q))}{\exists(x,\forall(r,q))}$ and $\frac{\exists(ax,\exists(r,q))}{\exists(x,\exists(r,q))}$ are full form of $\frac{\exists(ax,\beta(r,q))}{\exists(x,\beta(r,q))}$. Figure 3 and Figure 4 shows that derivations of sentences from a sentence or sentences in the language of the logic. “If $\Gamma \vdash \varphi$, then $\Gamma \vdash \psi$” is indicated by the arrows. The arrows do not work reverse direction.

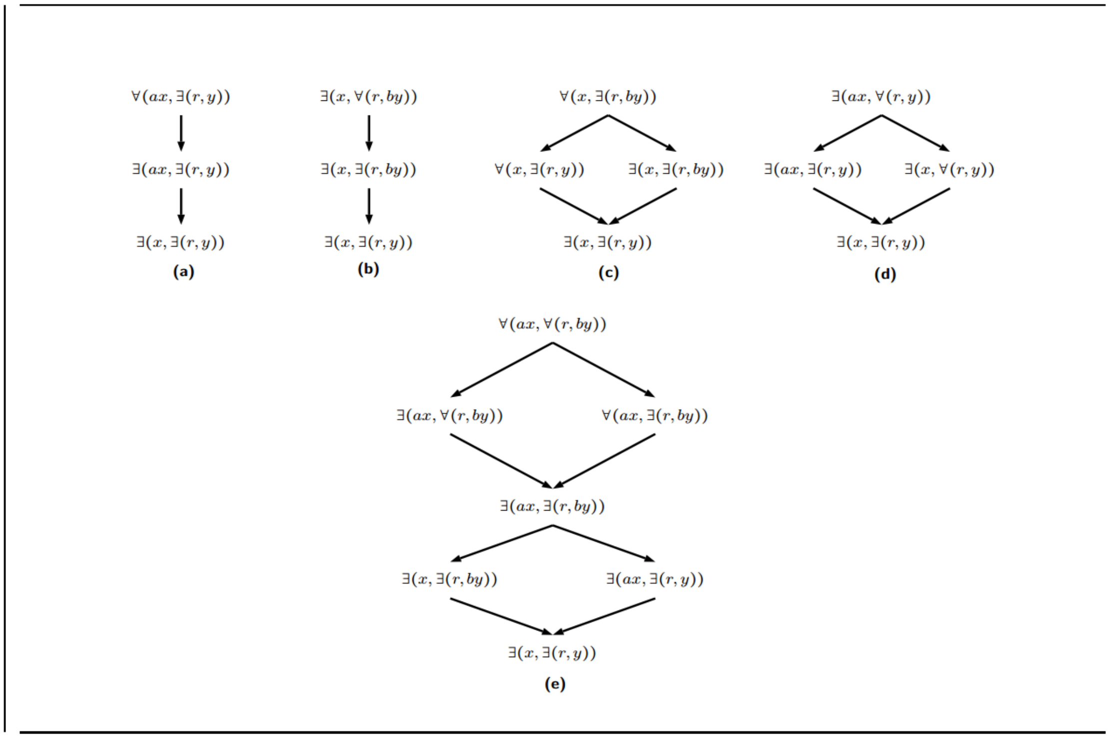

Fig. 4. Derivation diagrams for rules (6), (7), (8), (9), (10).

One of the main problems in logic is an algorithm to tell if $\Gamma|-\varphi$ or not. When one wants to check whether $\Gamma|-\exists(x,\exists(r,y))$ or not. All the arrows in Figure 3 and Figure 4 may be checked in the worst-case scenario for derivations in the logic. A model construction which tests being an element of a set and being a subset of a set is desired to not check the derivations in the scenario.

3.1. Model Construction

Definition 3.1. *P is a set of noun variables (complex or basic nouns), $\forall$ and $\exists$ arequantifiers in language of the logic. A set $\vec{P}$ consists of elements which accepted quantifiers in the language as subscript of nouns.*

Example 3.2. *If* $P=\{x,y,ax\}$ *, then* $\vec{P}=\{x_\forall, y_\forall, ax_\forall, x_\exists, y_\exists, ax_\exists\}$

Definition 3.3. *For a set of sentences Γ, P_Γ is the set of nouns occurring in Γ, R_Γ is the set of binary terms in Γ, and $\vec{P}_\Gamma$ is the set of elements of P_Γ with their quantifiers.*

Example 3.4. $\Gamma=\{\forall(x,\exists(r_0,y)), \exists(ax,\exists(r_1,by)), \forall(cz,\forall(r_1,z))\}$.

$R_\Gamma=\{r_0,r_1\}$, $P_\Gamma=\{ax,x,cz,by,z\}$, $\vec{P}_\Gamma=\{x_\forall, cz_\forall, z_\forall, ax_\exists, y_\exists\}$

Definition 3.5. *We define an translation from $\vec{P}$ to $\mathsf{P}(\vec{P})$ as the following:*

$[\,]:\vec{P}\mapsto \mathsf{P}(\vec{P})$

$x_\forall \mapsto \{x_\forall, x_\exists\}$
$x_\exists \mapsto \{x_\exists\}$
$ax_\forall \mapsto \{ax_\forall, ax_\exists, x_\exists\}$
$ax_\exists \mapsto \{ax_\exists, x_\exists\}$

Definition 3.6. *We define two sets* $[\vec{P}_\Gamma] = \{[i]: \text{for i in } \vec{P}_\Gamma\}$ *and* $M^+ \subseteq \vec{P} \times \vec{P} \times R$.

Definition 3.7. Let Γ be a set of sentences and $\Gamma_{Vec} \subseteq [\vec{P}_\Gamma] \times [\vec{P}_\Gamma] \times R_\Gamma$. We define a translation from Γ to Γ_{Vec}.

$$\Upsilon_V : \Gamma \mapsto \Gamma_{Vec}$$
$$\alpha(p, \beta(r, q)) \mapsto ([p_\alpha], [q_\beta], r)$$

Please notice that the translation is an one to one correspondence.

Remark 3.8. *Notice that* $\Gamma_{Vec} \subseteq M^+$.

Definition 3.9. *Two elements* $([k_\alpha], [l_\beta], r_0)$ *and* $([p_\gamma], [q_\theta], r_1)$ *of* Γ_{Vec} *are in the same equivalence class, if* $k = ax$ *or* $k = x$ *and* $p = ax$ *or* $p = x$ *and* $l = by$ *or* $l = y$ *and* $q = cz$ *or* $q = z$ *and* $r_0 = r_1$ *where* x, y, z *are basic nouns and* a, b, c *are intersective adjectives.*

Remark 3.10. *Two elements in* M^+ *are in the same equivalence class, if the two elements that first two elements are represented by the same letters and last ones are the same. For instance,* $([p_\alpha], [q_\beta], r)$ *and* $([p_\gamma], [q_\theta], r)$ *are in the same equivalence class because first objects of two elements are denoted by* p*, second ones are* q *and last ones are* r.

Definition 3.11. *A down-set of element* $([k_\alpha], [l_\beta], r_0)$ *of* M^+ *is a set* $d^R_\Downarrow[([k_\alpha], [l_\beta], r_0)] = \{([p_\alpha], [m_\beta], r_1) : [p_\alpha] \subseteq [k_\alpha] \text{ and } [m_\beta] \subseteq [l_\beta] \text{and} r_0 = r_1\}$ *and also we define* $d^R_\Downarrow[M^+] = \{d^R_\Downarrow[i] : i \in M^+\}$.

3.2. Constructing steps of $\overset{\triangleright}{M}{}^+_\Gamma$ *from* M^+_Γ

The following steps will be applied for every element of M^+_Γ. Note that we have first set $M^+_\Gamma = \Gamma_{Vec} \cap M^+$ before applying the following steps.

1. If $(\beta, \{x_\exists, x_\forall\}, r) \in M^+_\Gamma$ and $ax \in P_\Gamma$, then add $(\beta, \{ax_\exists, ax_\forall\}, r)$ to M^+_Γ.
2. If $(\{x_\exists, x_\forall\}, \gamma, , r) \in M^+_\Gamma$ and $ax \in P_\Gamma$, then add $(\{ax_\exists, ax_\forall\}, \gamma, r)$ to M^+_Γ.
3. If $(\{ax_\exists\}, \beta, r) \in M^+_\Gamma$ or $(\{ax_\exists, ax_\forall\}, \beta, r) \in M^+_\Gamma$, then add $(\{x_\exists\}, \beta, r)$ to M^+_Γ.
4. If $(\beta, \{ax_\exists\}, r) \in M^+_\Gamma$ or $(\beta, \{ax_\exists, ax_\forall\}, r) \in M^+_\Gamma$, then add $(\beta, \{x_\exists\}, r)$ to M^+_Γ.
5. If $(\{x_\exists, x_\forall\}, \beta, r) \in M^+_\Gamma$ and $ax \in P_\Gamma$, then add $(\{ax_\exists, ax_\forall\}, \beta, r)$ to M^+_Γ.
6. Finally, the last step is to take $d^R_\Downarrow[M^+_\Gamma]$ as $\overset{\triangleright}{M}{}^+_\Gamma$.

Example 3.12. *For a given* $\Gamma = \{\forall(x, \exists(r_0, c\,y)), \exists(a\,x, \forall(r_0, d\,y)), \forall(k, \exists(r_1, c\,l)), \exists(e\,x, \exists(r_2, b\,k))\}$,

$P_\Gamma = \{x, ax, cy, dy, k, cl, e\,x\}$,

$\Gamma_{Vec} = \{(\{x_\forall, x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0), (\{ax_\exists, x_\exists\}, \{dy_\forall, dy_\exists, y_\exists\}, r_0), (\{k_\forall, k_\exists\}, \{cl_\exists, l_\exists\}, r_1), (\{e\,x_\exists, e_\exists\}, \{bk_\exists, k_\exists\}, r_2)\}$

$\overset{\triangleright}{M}{}_\Gamma^+$ is composed of all elements in Table 3 and Table 4. The sign $\Downarrow$ indicates the sentences that can be derived from the sentence next to in the figures.

$(\{x_\forall, x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0)\ \Downarrow$	$(\{ax_\exists, x_\exists\}, \{dy_\forall, dy_\exists, y_\exists\}, r_0)\ \Downarrow$
$(\{x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0)$	$(\{ax_\exists, x_\exists\}, \{dy_\exists, y_\exists\}, r_0)$
$(\{x_\forall, x_\exists\}, \{y_\exists\}, r_0)$	$(\{x_\exists\}, \{dy_\forall, dy_\exists, y_\exists\}, r_0)$
$(\{a\,x_\forall, a\,x_\exists\}, \{y_\exists\}, r_0)$ **since** $ax \in P_\Gamma$	$(\{x_\exists\}, \{dy_\exists, y_\exists\}, r_0)$
$(\{a\,x_\exists\}, \{y_\exists\}, r_0)$	$(\{ax_\exists, x_\exists\}, \{y_\exists\}, r_0)$
$(\{a\,x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0)$	$(\{x_\exists\}, \{y_\exists\}, r_0)$
$(\{x_\forall, x_\exists\}, \{y_\exists\}, r_0)$	
$(\{x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0)$	
$(\{x_\exists\}, \{y_\exists\}, r_0)$	
$(\{e\,x_\forall, e\,x_\exists\}, \{y_\exists\}, r_0) \Downarrow$ *due to* $e\,x \in P_\Gamma$	

Table 3. Applying the constructing steps to $(\{x_\forall, x_\exists\}, \{c\,y_\exists, y_\exists\}, r_0)$ and $(\{ax_\exists, x_\exists\}, \{dy_\forall, dy_\exists, y_\exists\}, r_0)$

$(\{k_\forall, k_\exists\}, \{cl_\exists, l_\exists\}, r_1)\ \Downarrow$	$(\{e\,x_\exists,\ x_\exists\}, \{bk_\exists, k_\exists\}, r_2)\ \Downarrow$
$(\{k_\exists\}, \{cl_\exists, l_\exists\}, r_1)$	$(\{x_\exists\}, \{bk_\exists, k_\exists\}, r_2)$
$(\{k_\forall, k_\exists\}, \{l_\exists\}, r_1)$	$(\{e\,x_\exists, x_\exists\}, \{k_\exists\}, r_2)$
$(\{k_\exists\}, \{l_\exists\}, r_1)$	

Table 4. Applying the constructing steps to $(\{k_\forall, k_\exists\}, \{cl_\exists, l_\exists\}, r_1)$ and $(\{e\,x_\exists, x_\exists\}, \{bk_\exists, k_\exists\}, r_2)$

Definition 3.13. $[p_\alpha] \subseteq [k_\alpha]$ and $[m_\beta] \subseteq [l_\beta]$ and $r_0 = r_1$ *iff* $([p_\alpha], [m_\beta], r_1) \subseteq ([k_\alpha], [l_\beta], r_0)$

Theorem 3.14. $\Gamma \mid\!- \alpha(p, \beta(r, q))$ *iff* $([p_\alpha], [q_\beta], r) \in \overset{\triangleright}{M}{}_\Gamma^+$, in other words,

$\mathsf{M}_R = (M_R, [[\ ]]) :\Leftrightarrow \mathsf{M}_R = (\overset{\triangleright}{M}{}_\Gamma^+, \in)$.

Proof 3.14. We will prove the theorem on noun complexity. Proofs for sentences having universal quantifiers with only basic nounswere already given in $\mathsf{R}(\forall,\exists)$. Also, derivations of those sentences from a set of sentences are independent on existence of any other forms of sentences with or without adjectives. On the other hand, $\overset{\triangleright}{M}{}_\Gamma^+$ in $\mathsf{R}(\forall,\exists,\mathsf{IA})$ is a super set of $\overset{\triangleright}{M}{}_\Gamma^+$ in $\mathsf{R}(\forall,\exists)$. We will prove the theorem considering those situations.

($\Rightarrow$) Supposing $\Gamma \mid\!- \alpha(p, \beta(r, q))$, we will show that $([p_\alpha], [q_\beta], r) \in \overset{\triangleright}{M}{}_\Gamma^+$.

Case 1: $\Gamma \mid\!- \forall(ax, \forall(r, by))$ .If $\forall(ax, \forall(r, by)) \in \Gamma$,then $(\{ax_\forall, ax_\exists, x_\exists\}, \{b\,y_\forall, b\,y_\exists, y_\exists\}, r) \in M_\Gamma^+$, therefore, $(\{ax_\forall, ax_\exists, x_\exists\}, \{b\,y_\forall, b\,y_\exists, y_\exists\}, r) \in \overset{\triangleright}{M}{}_\Gamma^+$. Suppose $\forall(ax, \forall(r, by)) \notin \Gamma$ and $\Gamma \mid\!- \forall(x, \forall(r, y))$

and $a\,x \in P_\Gamma$ and $b\,y \in P_\Gamma$. We know that "if $\Gamma \mid\!- \forall(x, \forall(r, y))$, then $(\{x_\forall, x_\exists\}, \{y_\forall, y_\exists\}, r) \in M_\Gamma^+$" by Proof 2.15. $(\{ax_\forall, ax_\exists, x_\exists\}, \{b\,y_\forall, b\,y_\exists, y_\exists\}, r)$ is added to M_Γ^+ from the construction (1) and (2). Finally, $(\{ax_\forall, ax_\exists, x_\exists\}, \{b\,y_\forall, b\,y_\exists, y_\exists\}, r) \in \overset{\triangleright}{M}{}_\Gamma^+$.

Please note that it is hold for all derivable sentences from $\forall(ax, \forall(r, by))$ since all down sets of $(\{ax_\forall, ax_\exists, x_\exists\}, \{b\,y_\forall, b\,y_\exists, y_\exists\}, r)$ are contained by the construction.

Case 2: $\Gamma \mid\!- \exists(ax, \forall(r, by))$. If $\exists(ax, \forall(r, by)) \in \Gamma$, it is clear. Otherwise, there is a proof tree whose root is $\exists(ax, \forall(r, by))$. There are some cases for this derivation as the follows:

(a) If $\Gamma \mid\!- \forall(ax, \forall(r, by))$, we proved and mentioned it in Case 1.

(b) If $\Gamma \mid\!- \forall(x, \forall(r, by))$ and $ax \in P_\Gamma$, then $\Gamma \mid\!- \forall(ax, \forall(r, by))$ again.

(c) If $\Gamma \mid\!- \forall(ax, \forall(r, y))$ and $by \in P_\Gamma$, then $\Gamma \mid\!- \forall(ax, \forall(r, by))$ again (by Case 1).

(d) If $\Gamma \mid\!- \forall(x, \forall(r, y))$ and $ax, by \in P_\Gamma$, then $\Gamma \mid\!- \forall(ax, \forall(r, by))$ again (by Case 1).

Case 3: $\Gamma \mid\!- \forall(ax, \exists(r, by))$ is routine.

Case 4: $\Gamma \mid\!- \exists(ax, \exists(r, by))$ is routine.

Case 5: $\Gamma \mid\!- \exists(x, \exists(r, y))$. If $\exists(x, \exists(r, y)) \notin \Gamma$, there are possibly an awful lot of proof trees whose roots are $\exists(x, \exists(r, y))$ as can be seen in Figure 4. Starting the proofs as we mentioned, for all sentences which derive $\exists(x, \exists(r, y))$ are hold. If no sentences of Γ derives $\exists(x, \exists(r, y))$ except itself, it contradicts our $\exists(x, \exists(r, y)) \notin \Gamma$. Hence, if there exists at least one sentence which derives $\exists(x, \exists(r, y))$, then $([x_\exists], [y_\exists], r)$ must be in $\overset{\triangleright}{M}{}_\Gamma^+$.

Other proofs are routine.

$(\Leftarrow)$ We will show that $\Gamma \mid\!- \alpha(p, \beta(r, q))$ supposing $([p_\alpha], [q_\beta], r) \in \overset{\triangleright}{M}{}_\Gamma^+$.

If any $([p_\alpha], [q_\beta], r)$ in Γ_{Vec}, the proof is easy. Otherwise, we will use the down-set definition and property of one to one correspondence of Γ_{Vec}.

Let be $([ax_\forall], [by_\forall], r)$ in $\overset{\triangleright}{M}{}_\Gamma^+$. Suppose that $([ax_\forall], [by_\forall], r) \notin \Gamma_{Vec}$ (otherwise, $\forall(ax, \forall(r, by)) \in \Gamma$, therefore, $\Gamma \mid\!- \forall(ax, \forall(r, by))$). Then there is a $([p_\alpha], [q_\beta, r])$ where $([ax_\forall], [by_\forall], r)$ is a an element of $d_\Downarrow^R[([p_\alpha], [q_\beta, r])]$. So, $([p_\alpha], [q_\beta], r)$ must be in Γ_{Vec} since Υ_V is an one to one correspondence. Hence, $\alpha(p, \beta(r, q)) \in \Gamma$. Finally, $\alpha(p, \beta(r, q))$ follows $\alpha(p, \beta(r, q))$. Other proofs are routine.

Theorem 3.15. $\Gamma \mid\!- \varphi$ *iff there exists at least one* ψ *such that* $[\varphi] \subseteq [\psi]$ *in* $\overset{\triangleright}{M}{}_\Gamma^+$.

Proof 3.15. We saw that there is at least one upper set of φ to derive it from Γ or a sentence ψ due to the definitions Υ_V and down-sets in the sufficient condition of Theorem 3.15.

Corollary 3.16. *Let* Γ *be set of sentences in* $\mathsf{R}(\forall, \exists)$. $(M_\Gamma, [[\ ]])$, $(\overset{\triangleright}{M}{}_\Gamma^+, \in)$ *and* $(\overset{\triangleright}{M}{}_\Gamma^+, \subseteq)$ *are equivalent models.*

Corollary 3.17. *Let* Γ *be set of sentences in* $\mathsf{R}(\forall, \exists)$. $(M_\Gamma, [[\ ]])$, $(\overset{\triangleright}{M}{}_\Gamma^+, \in)$ *and* $(\overset{\triangleright}{M}{}_\Gamma^+, \subseteq)$ *are equivalent models.*

4. Conclusion

This paper has presented two logical systems and their set-theoretic semantics. The smaller system consists of transitive verbs and quantifiers. The bigger system is an extension of the small one which is restricted to intersective adjectives. The logical systems have three equivalent set-theoretic models. $(\overset{\triangleright}{M}{}^{+}_{\Gamma}, \in)$ and $(\overset{\triangleright}{M}{}^{+}_{\Gamma}, \subseteq)$ provide simplicity for checking derivability and non-derivability of a sentence from a set of sentences and also truth and falsity of a sentence in models of the logics because the models are built on the idea of equivalence class, being elements of a set and also testing whether a subset or not.

We hope that logico-linguists, applied and theoretical computer scientists, and pure and applied logicians might be interested in results in this paper.

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Reflexive Games in Management

Andrew Schumann

University of Information Technology and Management
in Rzeszow, Poland

e-mail: andrew.schumann@gmail.com

Abstract:
In this paper reflexive games are defined as a way to act beyond equilibria to control our opponents by our hiding motives. The task of a reflexive game is to have the opponent's actions become transparent for us, while our actions remain obscure for the competitor. In case a reflexive game is carried out between agents belonging to the same organisation (corporation, company, institute), success in a reflexive game can be reached by a purposeful modification of some components of a controlled system. Such a modification for the guaranteed victory in a reflexive game is called reflexive management. This kind of management uses reflexive games to control a knowledge structure of agents in a way their actions unconsciously satisfy the centre's goals.
Keywords: reflexive game, reflexive management, speech competence, discourse community.

1. Introduction

One of the directions in pragmatic studies is presented by reflexive games. For the first time, Vladimir Lefebvre formulated reflexive games assuming many reflexion levels [8], [9], [10]. Reflexive games are understood as an extension of epistemic games [2]. The game-theoretic mathematics for the early ideas of Lefebvre has been developed by Dmitry Novikov and Aleksandr Chkhartishvili [3], [4], [5], [6]. In this paper I appeal to the approach to reflexive games proposed in [12], [13], [14]. This approach is unconventional and assumes cellular-automatic calculations. First, I define the context of reflexive games (section 2) and show why in reflexive games there are no conventional equilibria. Then I introduce the notion of reflexive games in accordance with the ideas of [12], [13], [14] (section 3). Further, I show how we can apply reflexive games in the management practice within the so-called reflexive management (section 4). Finally, I consider the role of reflexive management in discourse communities (section 5).

2. Enemies and Games Beyond Nash Equilibria

In the Austrian school of economics it is supposed that the simple mutually advantageous interchange is always possible. In the words of game theorists, this means that the Nash equilibrium is always possible. For example, I produce apples, my neighbour produces pears. Nevertheless, I like pears and my neighbour apples. Then the Nash equilibrium is reached by the mutually advantageous interchange of apples and pears, for example by using the formula: one apple for one pear and vice versa. During the interchange each actor is rational, knows the set of game players, the goal functions and the admissible set of actions of all players, and also knows the set of possible values of states of affairs. Such knowledge can be reached, in particular by a public communication of appropriate information of all agents met at one place. This communication allows them to find the Nash equilibrium, a simple mutually advantageous interchange. In the Nash equilibrium there is a parity of reflexive relations of all players. On the one hand, both actors have autonomy, different goal functions and, on the other hand, both help each other to reach goals by means of a mutually advantageous interchange, knowing everything about each other. As the first approximation, the stock exchange is an example of such an equilibrium.

Let us suppose now that rational agents are our enemies. They do not wish to help us to reach the equilibrium of our goal functions by means of an interchange. In every possible way they hinder us from having the usual interchange with other players (for example, they use dumping practices so that we will go bankrupt). In this case the Nash equilibrium cannot be reached. We cannot wait for a simple mutually advantageous interchange of goods.

Competitiveness complicates any strategy of reaching a maximal guaranteed payoff. We should already deal with reflexive games in order to evaluate other actors, for example, to reconstruct their goal functions taking into account circumstances in which they can try to delude their environment concerning the original motives of their acts. The main task of reflexive games is to hide true motives and goals, not to be transparent for others, but to know everything important about them. Let us consider Edgar Allen Poe’s example of reflexive games:

> I knew one [schoolboy] about eight years of age, whose success at guessing in the game of ‘even and odd’ attracted universal admiration. This game is simple, and is played with marbles. One player holds in his hand a number of these toys, and demands of another whether that number is even or odd. If the guess is right, the guesser wins one; if wrong, he loses one. The boy to whom I allude won all the marbles of the school. Of course he had some principle of guessing; and this lay in mere observation and measurement of the astuteness of his opponents. For example, an arrant simpleton is his opponent, and, holding up his closed hand, asks, ‘are they even or odd?’ Our schoolboy replies, ‘odd,’ and loses; but upon the second trial he wins, for he then says to himself, ‘the simpleton had them even upon the first trial, and his amount of cunning is just sufficient to make him have them odd upon the second; I will therefore guess odd;’—he guesses odd, and wins. Now, with a simpleton a degree above the first, he would have reasoned thus: ‘This fellow finds that in the first instance I guessed odd, and, in the second, he will propose to himself, upon the first impulse, a simple variation from even to odd, as did the first simpleton; but then a second thought will suggest that this is too simple a variation, and finally he will decide upon putting it even as before. I will therefore guess even;’—he guesses even, and wins. Now this mode of reasoning in the schoolboy, whom his fellows termed ‘lucky,’—what, in its last analysis, is it?
>
> ‘It is merely,’ I said, ‘an identification of the reasoner's intellect with that of his opponent.’
>
> ‘It is,’ said Dupin; and, upon inquiring, of the boy by what means he effected the thorough identification in which his success consisted, I received answer as follows: ‘When I wish to find out how wise, or how stupid, or how good, or how wicked is any one, or what are his thoughts at the moment, I fashion the expression of my face, as accurately as possible, in accordance with the expression of his, and then wait to see what thoughts or sentiments arise in my mind or heart, as if to match or correspond with the expression.’ This response of the schoolboy lies at the bottom of all the spurious profundity which has been attributed to Rochefoucault, to La Bougève, to Machiavelli, and to Campanella (Edgar Allen Poe, *The Purloined Letter*).

In this example, the schoolboy has success at guessing in the game of ‘even and odd,’ because he considers it not as simple guessing, but as a reflexive game and correctly defines two kinds of gamers: ‘an arrant simpleton’ who permanently changes the strategy upon different trials, and a ‘simpleton a degree above the first’ who uses the same strategy upon different trials for cheating (cheating since the game is understood by gamers as pure guessing). In other words, the game of ‘even and odd’ assumes two levels of reflexion: the first level consisting in using casually different strategies, the second consisting in using the same strategies and in avoiding casual choices of strategies. Different people with different intelligent abilities play at different reflexive levels. How many levels can exist in reflexive games in all?

Let us imagine a nightmare. A huge monster runs after us and its speed is obviously faster. We face two caves. The monster does not have time to see which of the caves we choose. The first cave is twisting and the second is a straight line as a tunnel. It is an example of a reflexive game. I select a cave, assuming which cave the monster will choose. Let us consider the possible levels of reflexion:

- *The reflexion of zero level*: I do not think that the animal thinks, and the animal does not think that I think. I choose the twisting cave, my arguments are as follows: in the twisting cave any speed is reduced and I have a possibility to escape from the monster; in running through it I will not be in the monster’s sight and my further actions will not be known by the animal. For the monster the reflexion of zero level can mean a choice of the direct cave, as it is easier to run through this cave.
- *The first-level reflexion*: I think that the animal thinks, and the animal thinks that I think. Formally: $Think_A(Think_B)$ and $Think_B(Think_A)$, where agent A is me and agent B is the monster. The monster at this level of reflexion will run through the twisting cave. It already tries to predict my behaviour and my choice of cave. I also will run through the twisting cave, as I know that at the zero level of reflexion the animal chooses the direct cave.
- *The second-level reflexion*: I think that the animal thinks, thinking that I think, and the monster thinks that I think, thinking that the monster thinks. Formally: $Think_A\,(Think_B\,(Think_A))$ and $Think_B\,(Think_A\,(Think_B))$. Having selected the twisting path, I generally did not evaluate the mental abilities of the monster to deceive me. I assumed that it is able only to commit direct actions and is not able to deceive. However, this assumption can become false. The monster can predict my actions in order to understand what I think of it. The second level of reflexion is that I assume that the monster wishes and is able to predict my actions as an intelligent being. Then I should choose the direct cave. My arguments: any intelligent being selects the twisting cave, because it is easier to be rescued in it, but such logic is transparent for any rational agent, the same for the monster, if it is rational. At the second level of reflexion I try to predict the actions of the monster recognising that it considers me an intelligent being and I try to act not in the way it expects. The monster at the second level of reflexion also runs through the straight cave.
- *The third-level reflexion*: I think that the animal thinks, thinking that I think, thinking that I think. $Think_B\,(Think_A\,(Think_B\,(Think_A)))$ and $Think_A\,(Think_B\,(Think_A\,(Think_B)))$. At the second level of reflexion I detect the monster’s ability (as a reflexive player of the first level) to predict my behaviour, but I have not yet assumed that the monster itself can have the ability of reflexion of the second level and it can act not in the way I expect. At the second level of reflexion I expected that it should run through the straight cave. Nevertheless, the animal can know itself about this by my waiting, therefore at the third level of reflexion I select the twisting cave. My arguments: any being capable of an elementary reflexion of the second level will select the direct cave, knowing that only the most naive rational agents select the twisting cave. We wish to act unpredictably for rational agents, therefore we choose again the twisting cave. But at the third level of reflexion the monster will run also through the twisting cave. It also assumes that we are capable of deceit.
- *The fourth-level reflexion*:
$Think_A(Think_B(Think_A(Think_B(Think_A))))$, $Think_B(Think_A(Think_B(Think_A(Think_B))))$.

However, my logic with the desire to be unpredictable can also be transparent for the monster. Consequently, I cannot be rescued again through the twisted cave. I should choose the direct cave. Which cave should I run through then? Which cave will the animal run through?

In this game of choosing the caves I lose and the monster wins, if $n > 0$ and my level of reflexion and the monster's level of reflexion are the same number n. I win, if the monster's level of reflexion is n and my level of reflexion is $n + 1$.

This example with the monster shows that reflexion levels can be an arbitrary natural number. If I do not know the monster's mental abilities, I cannot select the level of reflexion upon which I should act. Then I will stand before both caves without the possibility of finding any true level of reflexion. In this time the monster will overtake me and eat me. It is an example of the *reflexivity paradox,* i.e. the impossibility of defining a true level of reflexion for a successful interaction with competitors.

Another example of a reflexive game when the reflexivity paradox is possible is hide-and-seek. The first actor hides in one of several rooms with different lighting, and another agent should select that room where he will search for the first actor. The degree of lighting is known by both agents. The strategies of the agents are as follows. The second actor, who searches, rather prefers to search where it is lighter (easier to find). On the contrary, the first actor, who hides, rather prefers to hide in dark rooms, because there are more chances to be undiscovered. It is a zero level of reflexion for both agents. The increase of reflexion degrees means that it becomes clear to the agent that it is clear also to his opponent, etc. If I do not know the mental abilities of the opponent, the paradox of reflexivity will hold. Then I cannot select the rooms in which it is more preferable to search (hide). At the same time, the first actor, who hides, wins, if $n > 0$ and his level of reflexion is $n + 1$, when his opponent's level of reflexion is n. The second, who seeks, wins if $n > 0$ and his level of reflexion, n, is the same as his opponent's level of reflexion.

It is obvious that if there are no data about a competitor's mental abilities at all, I can act at the zero level of reflexion, i.e. I can ignore the competitor's intellectual possibilities in his play against me. If there is an occasion to guess their mental abilities, I select reflexion level n with respect to the opponent's abilities to play in reflexive games and my possibilities of winning.

If at least one agent selects a game strategy assuming a non-zero level of reflexion, then this game is called a *reflexive game*. Its essence consists in finding the level of reflexion n of the competitor ($n > 0$) to move onto reflexion level n (if I have advantages at the equal level of reflexivity) or $n + 1$ (if I have no advantages) and to act on the basis of the given level. The task of a reflexive game is to have the opponent's actions become transparent for us, while our actions remain obscure for the competitor.

3. On the Notion of Reflexive Game

Let us notice that in our everyday life we permanently face reflexive games. Thereby gamers can follow different levels of reflexion upon different trials of the same game. A reflexion level is not constant. It is a dynamic index. Accordingly, the victory in a reflexive game is determined by who has managed in most cases to be in dialogues at a level of reflexion n or n+1 while the interlocutor remained at level n. The more difficult the reflexive game, the more information we should give about ourselves to uncover all motivations and all predispositions of the interlocutor.

There are too many examples of daily reflexive games. Let us consider relationships in a family. Does a husband or a wife have a priority in a reflexive game? Who should be the leader in a family? Are equal relations possible? Or consider relationships with subordinates. Should reflexive games be carried out in relations with subordinates?

Rules in reflexive games depend on the following parameters:

- *number of agents* (a leader of a group is presented by an agent who is capable in dialogues of being at reflexion level n or n+1 while all other interlocutors remain at level n upon major trials of the same game; notice that for each pair of agents the number n can be different);

- *preferences of agents* (different goal functions and dependences of their payoff on actions, e.g. when we know that each agent is interested in a maximisation of payoff and for this purpose (s)he commits a minimal set of certain actions, and for different agents this set can be different);
- *set of admissible actions of agents* (there are actions which are unacceptable for all in the group of agents, and there are actions which are expected or not expected by other agents, but these actions are admissible for the entire group of agents);
- *knowledge of agents* (at the moment of decision making agents should be informed, probably falsely, about all preferences of other agents);
- *order of moves* (sequence of choices of actions, comprehensible to all in the group of agents).

Thus, preferences express what agents want, sets of admissible operations express what they can do, knowledge expresses what they know, and order of moves express when they select actions.

The larger the number of agents in a group, the more complex task to be a leader (to reach a victory in a reflexive game upon major trials). An elementary case is the game with two actors. Such games can be considered in the bimatrix form. So, the monster's run is a bimatrix reflexive game of the form (x, y), where x is my choice (0 is a straight line, 1 is a twisted cave), y is the monster's choice (0 is a straight line, 1 is a twisted cave). I win, if $x \neq y$, and the monster wins, if $x = y$. Values of x and y depend on the reflexion level. At an equal level of reflexion $x = y$ and the monster wins, as at the level of direct actions it has advantages. At reflexion level n for the monster and n+1 for us there is no Nash equilibrium.

A classical example of a bimatrix reflexive game is the Prisoner's Dilemma. Each of two prisoners can choose one of the following two actions: "to confess a crime" and "not to confess a crime." If both agents cooperate with the police, both are sentenced and the vector (1 year, 1 year) is their payoff. If the first confesses and the second does not, then the first goes free and the second is sentenced and the vector (goes free, 3 years) is their payoff. If the second confesses and the first does not, then (3 years, goes free). And if both do not confess, their punishment will be equal (2 years, 2 years).

In reflexive games we deal with an unlimited hierarchy of cognitive pictures. Let us consider a bimatrix game with agents i and j. Each of them can have their own picture about a state of affairs A. Denote these pictures by K_iA and K_jA respectively. *The first-order reflexion* (thoughts about pictures of the opponent) is expressed by means of *pictures of the second order* which are designated by K_jK_iA and K_iK_jA where K_jK_iA are pictures of agent j about pictures of agent i, K_iK_jA are pictures of agent i about pictures of agent j. *The reflexion of the second order* defines which pictures of one opponent are related to pictures of another opponent. At this level of reflexion pictures of *the third order* $K_iK_jK_iA$ and $K_jK_iK_jA$ are generated. And so ad infinitum. The collection of all pictures K_iA, K_jA, K_jK_iA, K_iK_jA, $K_iK_jK_iA$, $K_jK_iK_jA$ etc. makes an infinite hierarchy.

Definition 1. The reflexion of the agent i on the n-th level in bimatrix games is expressed by (n+1)-order knowledge operators $K_i^{n+1}A = K_iK_jK_i\ldots A$, where on the right side there are n+1 K_m-operators ($m = i, j$).

Let us consider two agents i and j and suppose that the reflexive game takes place on level n. This means that we have $K_i^{n+1}A$ and/or $K_j^{n+1}A$ which are understood as perlocutionary effects of illocutionary acts [15], [16] and satisfy requirements:

$$(K_iA \cap K_iB) \Rightarrow K_i(A \cap B); \tag{1}$$

$$K_i(A \cup B) \Rightarrow (K_iA \cup K_iB); \tag{2}$$

$$K_i(A \cup B) = (K_iA \cap K_iB); \tag{3}$$

$$A \subseteq B \Rightarrow K_iA \subseteq K_iB; \tag{4}$$

$$A \subseteq K_i A; \quad (5)$$

$$K_i K_i A = K_i A. \quad (6)$$

For more details see [12], [13], [14].

We know that $A \subseteq \ldots \subseteq K_j^n A \subseteq K_i^{n+1} A$ and $A \subseteq \ldots \subseteq K_i^n A \subseteq K_j^{n+1} A$. Therefore $K_i^{n+1} A \cap K_j^{n+1} A \neq \varnothing$.

Definition 2. The payoff of a reflexive game on the *n*-th level in accordance with $K_i^{n+1}A$ or $K_j^{n+1}A$ is called performative equilibrium of this game.

We have the following possibilities:

- both $K_i^{n+1}A$ and $K_j^{n+1}A$ are a performative equilibrium—this means that agents *i* and *j* are on the *n*-th level of reflexion, simultaneously;
- only $K_i^{n+1}A$ is a performative equilibrium (then we can take $K_j^{n+1}A = K_j^n A$) – this means that agent *i* stays on the *n*-th level of reflexion, but agent *j* stays on the $(n – 1)$-th level of reflexion;
- only $K_j^{n+1}A$ is a performative equilibrium (then we can take $K_i^{n+1}A = K_i^n A$) – this means that agent *j* stays on the *n*-th level of reflexion, but agent *i* stays on the $(n – 1)$-th level of reflexion.

For any reflexive game on the *n*-th level of reflexion we can build up a tree of graphs. Vertices of the tree correspond to real or phantom agents, participating in reflexive game. Branches of the tree simulate a mutual knowledge of agents on reflexion level *n*: if from (real or phantom) agent *i* there exists a path to agent *j*, then agent *j* is correctly informed about agent *i*. In this case $K_j^{n+1}A$ is a performative equilibrium. If both $K_i^{n+1}A$ and $K_j^{n+1}A$ are a performative equilibrium of the same game, then an appropriate tree has a loop.

In a reflexive game on level *n* it is important for agent *i* that $K_i^n A \subseteq K_j^{n+1} A$ holds, because it means that agent *i* has really corresponded to level *n*. Correctly defining the level *n* of reflexion implies a victory in a game. Let us consider the game of two brokers to show how it is sophisticated sometimes to define *n*. Two brokers at a stock exchange have appropriate expert systems which have been used for the support of decision making. The network administrator illegally copied both expert systems and sold each broker an expert system of his opponent. Then he tries to sell each of them the following information: “Your opponent has your expert system.” Then the administrator tries to sell the information: “Your opponent knows that you have his expert system,” etc. How should brokers use the information received from the administrator and also what information on what iteration is essential? Theoretically, reflexive level *n* can be any natural number.

4. On the Notion of Reflexive Management

Any everyday dialogue can be considered a reflexive game. Each person, speaking those or other things, tries to obtain something from us. We always try to understand the motives (s)he has for talking to us. Do they (s)he wish only to learn something from us or to influence us? How exclusive is the message which (s)he utters? Will we begin to know more on the topic after the talk? Is (s)he sincere? How sincerely does (s)he express for us his/her strategy of creative reasoning?

Emotions, which are expressed in illocutions, are one of the main forms of reflexion. The interchanging of emotions is always a reflexive game, a method of manipulation of others. The character played by Sharon Stone in *Basic Instinct* (the 1992 movie) shows reflexive abilities in emotional management. How transparent are her emotions? Are we capable of winning emotionally in games with her or at least of reaching an emotional consensus? Her emotions are not at all transparent for us as are the emotions of coaching trainers who better know strategies of management struggle and overcome us in any reflexive game.

Insufficient knowledge (lack of common knowledge $K_i^{n+1}A$) of agent *i* on reflexion level *n* leads to an actual vector of actions on reflexion level *n* that can differ from a vector expected by agent *i*. For reaching a performative equilibrium it is expedient to follow the following assumptions:

1. The finite number of real and phantom agents participate in a reflexive game.
2. Equally informed agents select identical actions according to reflexion level *n*.
3. The rational behaviour of agents consists in that each of them aspires to maximise a goal function by a choice of appropriate actions, predicting which actions other agents will choose as rational agents from the point of view of knowledge of reflexion level *n* about other agents.

In case a reflexive game is carried out between agents belonging to the same organisation (corporation, company, institute), success in a reflexive game can be reached by a purposeful modification of some components of a controlled system. Such a modification for the guaranteed victory in a reflexive game is called *reflexive management*. The principal kinds of reflexive management are as follows:

- *institutional management* (modification of admissible sets of actions of all groups of agents);
- *motivational management* (modification of goal functions of concrete agents);
- *informational management* (modification of information which agents use in decision making).

Informational management refers to the following kinds:

- *informational regulating* (purposeful influence on information about states of affairs);
- *expert management* (purposeful influence on information about models of decision making);
- *active prognosis* (purposeful spread of information about future values of parameters depending on states of affairs and actions of actors).

The task of reflexive management is formulated as follows: a controlling organ creates a knowledge structure of agents in a way such that a performative equilibrium satisfies the centre's goals (maximally favourable for this centre.)

Management of an opponent's decision-making can be carried out by means of suggestions to him/her of some foundations from which (s)he could logically infer decisions favourable to us. Such a process of suggesting foundations for an opponent's decision-making is called *reflexive management*. Reflexive management can be performed by means of saying false information about a state of affairs (creation of false objects), by means of suggesting an opponent's purposes (provocations and intrigues, acts of terrorism and ideological diversions), or by means of suggesting decisions (false advice).

5. Reflexive Management in Discourse Communities

A reflexive game is probable only in a case where agents can reach *performative equilibrium* — they can act concordantly at reflexion levels $n > 0$. This condition is fulfilled in the case where there are mechanisms of intercommunication broadly agreed upon among people. These mechanisms have been preserved within an appropriate discourse community (*Kommunikationsgemeinschaft*) [1] shared by members with a suitable degree of discoursal expertise (i.e. members possess one or more genres in the communicative furtherance of its aims and know a specific lexis) and with a degree of relevant content to provide information and feedback.

Any discourse community represents a group of people who are in permanent interactions with each other and exchange performative propositions. This community is self-organised. Due to common discourse it can reach an *informational equilibrium*, and also a parity of creative reasoning as well as emotional consensus in interchanging performative propositions.

Members of a discourse community have a common speech competence (*Sprachkompetenz*) [1], sufficient for interactions. Let us recall that speech competence as such is comprehended neither by members of a discourse community, nor by outside agents, but its possession is a necessary condition for entry into an appropriate discourse community. Speech competence is understood as the knowledge and ability to use language in accordance with different contexts. Thus, modelling by speech competence is a key notion for managing a discourse community.

Any stable group of people united by joint interests is a discourse community. It is presented by appropriate forms of consolidation. In some cases the emotional community of its members leads to the appearance of corporate ethics, i.e. to sharing values and priorities. Systems of sanctions, such as blame or elimination from the group, are possible also.

A discourse community can be transformed into an appropriate social institute. Then group interests are formulated in an explicit form. Common centres of decision making appear and individual acts of activity are coordinated with joint plans. Within social institutes the discourse community becomes a hierarchical system. Its degree of complexity is presented by the opacity of the decision-making mechanism (the closeness of the mechanism for how creative reasoning is constructed, the hiding of a maximal reflexion level of the centre). The different degrees of openness of a social structure is possible also.

A discourse community is a multiagent system whose participants have the ability to act, including a freedom to choose states and strategies of speech behaviour. Besides a possibility of choice of activity schemata, members of a discourse community bear characteristic interests and preferences which can contradict interests and preferences of other members.

In any multiagent system we assume that there is a collection of subjects and objects which are units of the system, but they can be different by the nature: rational, irrational, intelligent, phantom agents, etc. Among these items there is a family of informational, controlling and other links, including subordination relations and distributions of the right to make decisions.

Rules, according to which the criteria of interaction effectiveness (performative equilibrium) are made, define which agents are rational or irrational. The dynamics of a system depends on the variety of preferences of agents and on the ways of consensus in the context of control actions. The order of system functioning can be revealed by detecting a sequence of process data and a choice of strategies made by system members. Thus, the functioning order depends on how often different strategies are chosen.

The life engineering cycle has the following stages: (1) design (concept design, detailed design, validation), (2) realization (plan manufacturing, manufacture, test), service (sell and deliver, use, maintain and support, dispose). By analogy it is possible to point out a performative cycle of a discourse community: (1) showing joint interests, (2) definitions of appropriate forms of performative equilibriums to reach joint interests, (3) implementation and realizing corresponding performative equilibriums, (4) loss of joint interest. For example, the performative cycle of a club of salsa fans consists of the stages: acquaintance of several salsa fans, finding a place for regular meetings (for example, in a bar), realisation of meetings, acceptance of new members, closing.

In hierarchic discourse communities (for example, in social institutes) the dialogue with the centre can be considered a reflexive game. The more difficult the hierarchic multiagent system, the higher the order of reflexion of the centre. The game task consists in explicating performative cycles of the system in order to uncover the centre’s mechanisms of planning and stimulation and, then, to involve the centre in a reflexive game, having the order of reflexion sufficient for obtaining victory in this reflexive game.

Centralised systems are a variety of hierarchic ones. Their disadvantage is in that subordinates ignore a part of their obligations and avoid full responsibility, meaning that their manager is completely responsible. The effectiveness of management for such a business is insignificant. The manager is too busy dealing with routine, and the employees half idle, expecting visit from the manager. In such systems any reflexive game with the centre has a low reflexion level, therefore performative equilibriums do not assume a high cognitive and emotional consensus of participants.

The system, in which there is a delegation of powers, where the decision-making process is distributed throughout the entire system of management, is more rational than the centralised system. The higher tasks of organisation are divided into many more detailed tasks for which solutions specific employees are responsible. Hence, each employee (1) surely knows what action (s)he is responsible for; (2) knows what resources (s)he can use independently and in what cases (s)he can ask the manager about additional resources; (3) knows how outcomes of activity are evaluated and knows the method of reward for success. These conditions provide the system with complex reflexive games making the system more stable performatively.

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Russian Orthodoxy and the Western World

Basil Lourié born in 1962, St Petersburg, Russia. PhD and Dr habil. in philosophy. Editor in Chief of the *Scrinium. Revue de patrologie, d'hagiographie critique et d'histoire ecclésiastique* and of its two supplement series: *Orientalia Judaica Christiana* and *Scripta ecclesiastica*. Interests: Patristics, Christian Origins, Second Temple Judaism(s). Books in Russian: 1. *Gregory of Nyssa, De hominis opificio* (1995; 2nd ed., 2000) (tr. and scholarly commentary). 2. *Vocation of Abraham. The Idea of the Monasticism and Its Realization in Egypt* (2000). 3. *History of the Byzantine Philosophy. Formative period* (2006) ([some fragments on the publisher's web-site http://www.axioma.spb.ru/z_byz_phil/intro.htm]). Serbian translation (2010). 4. *Introduction to the Critical Hagiography* (2009). 5. *Russian Orthodoxy between Kiev and Moscow* (2009; 2nd ed., 2010).

Tudor Petcu: It is well-known the fact that the Russian culture has always played a very important role in western societies and I would say that Westerns discovered it much better after the Second World War because of the Russian immigrants. So, please, explain us what does it mean in fact the Russian identity for the western cultures and of course for their development.

Basil Lourié: I think that the most important was not the second wave of Russian emigration (after the WWII), but the first one (in 1917 – 1922). The first wave discovered to the West a lot of Russian culture. But the second one was more Sovietized and not so useful for the West (with some exceptions, however: such as a small number of the Russian Catacomb Christians). The first Russian emigration discovered to the West Russia of “Tolstoevsky” and the Orthodoxy, sharply distinct from the Soviet Union. It became an impetus for the Western culture in some field (esp. of scholarship and religion) but not especially radical. I would prefer to avoid an overestimation of this impact.

Tudor Petcu: The best way to understand the Russian identity is probably Orthodoxy, the most ancient Christian tradition. Do you think that different Russian immigrants who were established in West helped its citizens to discover Orthodoxy in a deeper way? We shouldn't forget that many Westerns have chosen to become Orthodox.

Basil Lourié: Indeed, some influence of Russian emigration was sensible in this respect. At least, the Russian emigration turned out a bit more successful than the Greek one (which was also enormous after the Greek catastrophe in Asia Minor in 1922). However, there was always a problem with this Russian export-quality Orthodoxy: whether the Western convert will become Orthodox or Russophiles. ROCOR (Russian Church Outside Russia) destroyed with her own hands, in the late 1960s, a successful project of the European Orthodoxy (with the Western rite and services in different European languages) by its own Exarch of Western Europe St John of Shanhai and San-Francisco. I would say that the Orthodox mission was more successful in North America than in Europe. It was even more successful in popularisation of Eastern Patristics among the Western (especially Catholic) scholars.

Finally, I would not agree with the claim that the Orthodoxy is especially important for Russian identity. Our great Orthodox and nationalistic thinker, Constantine Leontiev, realised this fact (and, thus, asked: "Do we really need Russia non-monarchic and non-Orthodox?'). Russian identity is often understood as expressed in some Christian folklore: this is hardly a right opinion, and this folklore has certainly nothing to do with true Christianity).

Tudor Petcu: Would it be correct to say that the Russian Orthodox Church Outside of Russia has meant the rebirth of some western orthodox communities?

Basil Lourié: Such ideas were close to the hearts of some its members. The most known among them is St. John of Shanhai (+ 1966). But they always were a small minority within ROCOR. The majority of both people and bishops were seeking for a "Russian club".

Tudor Petcu: What would you say about the book written by Vladimir Moss, "The Fall of Orthodox England"? I make reference to his book because he is trying to highlight some very important aspects concerning the Russian Orthodox Church.

Basil Lourié: I think that it is a good book of vulgarization, useful for the first approaching to the topic. But I forgot what is said there about the Russian Church. Anyway, Vladimir wrote a large book "The Orthodox Church on the Crossroads", where his views are exposed in an elaborated way. I cannot say that I share all his views, although, indeed, I agree with him that the only real Church under the Soviet regime was the Catacomb Church.

Tudor Petcu: Which are the main important western countries where Russian Orthodoxy has known the strongest evolution?

Basil Lourié: U.S.A. and France.

Tudor Petcu: I could not forget about one of the most important orthodox monasteries in England, called Saint John the Baptist and located in Essex. This monastery is well-known especially because of Saint Father Sophrony who was Russian and I would like you to tell me how did manage his personality to influence the evolution of Orthodoxy in England.

Basil Lourié: Fr Sophrony became very popular after his 1952 book about the Athonite Startets (Elder) Siluan. Then, Fr Sophrony became an elder himself, which provoked some tension between him and then the head of the Moscow Patriarchate's local diocese Metropolitan Anthony (Bloom). This is why Fr Sophrony and his monastery turned out under Constantinople. Some of the modern Orthodox believe that Fr Sophrony was a genuine Starets and so, established an important spiritual centre. Some others think otherwise and, changing a little the words, paraphrase the title of his bestsellers How I see God as He is.

Tudor Petcu: Over the years I have had the privilege to make interviews with many Western Orthodox theologians and not long time ago, I have found out that there is also what we can call the Western Orthodox Church, reborn especially in France in 20th century. This rebirth was actually a result of Eugraph Kovalevsky's actions, an immigrant from Russia, whose main purpose was the resurrection of French and Western Orthodoxy. So, how would you describe his personality as a Russian Orthodox for a new era of Orthodoxy in West?

Basil Lourié: Eugraph Kovalevsky was the heart and the driving force of the project under the omophorion of St John of Shanhai, which I have mentioned above. He lived in an extremely aggressive milieu and was not always able to see the right path in such muddle. But his missionary zeal was absolutely justified. I strongly believe that, in Western Europe, the Orthodox faith must be wrapped with the Western rite.

Tudor Petcu: As we know, there are numerous Russian orthodox theologians who lived in West such as Vladimir Lossky or Sergei Bulgakov. Given the breadth and importance of their theological work, how did they influence, from your point of view, the western Christian theology, especially the catholic one?

Basil Lourié: There is, in the West, a narrow scholarly milieu of those who study Bulgakov, Florensky, and Vladimir Soloviev. Those scholars who are interested in Patristics normally do not read them. Thus, I doubt that there is any serious influence of Bulgakov (unlike Soloviev) on the Catholic and other Western theologians. Bulgakov influenced, however, his Russian opponents Georges Florovsky and John Meyendorff who, in turn, influenced Western scholarship immediately.

Vladimir Lossky, a disciple of Etienne Gilson and a strong opponent of Bulgakov, is a quite different story. He influenced both Russian patrologists such as Meyendorff but also the Western scholars directly. He could be considered as the founder of the present-day "Neopatristic Synthesis". But Lossky was inspired by the need to write against Bulgakov's"Sophiology. So, in this way, both Bulgakov and Lossky are of importance.